

Mathematica 11.3 Integration Test Results

Test results for the 254 problems in "4.7.1 (c trig)^m (d trig)^{n.m"}

Problem 8: Result more than twice size of optimal antiderivative.

$$\int \csc[2a + 2bx] \sin[a + bx] dx$$

Optimal (type 3, 14 leaves, 2 steps):

$$\frac{\operatorname{ArcTanh}[\sin[a + bx]]}{2b}$$

Result (type 3, 72 leaves):

$$\frac{1}{2} \left(-\frac{\operatorname{Log}[\cos[\frac{a}{2} + \frac{bx}{2}] - \sin[\frac{a}{2} + \frac{bx}{2}]]}{b} + \frac{\operatorname{Log}[\cos[\frac{a}{2} + \frac{bx}{2}] + \sin[\frac{a}{2} + \frac{bx}{2}]]}{b} \right)$$

Problem 10: Result more than twice size of optimal antiderivative.

$$\int \csc[2a + 2bx]^3 \sin[a + bx] dx$$

Optimal (type 3, 49 leaves, 5 steps):

$$\frac{3 \operatorname{ArcTanh}[\sin[a + bx]]}{16b} - \frac{3 \csc[a + bx]}{16b} + \frac{\csc[a + bx] \sec[a + bx]^2}{16b}$$

Result (type 3, 132 leaves):

$$\begin{aligned} & -\frac{1}{32b} \left(2 \cot[\frac{1}{2}(a + bx)] + 6 \operatorname{Log}[\cos[\frac{1}{2}(a + bx)] - \sin[\frac{1}{2}(a + bx)]] - \right. \\ & 6 \operatorname{Log}[\cos[\frac{1}{2}(a + bx)] + \sin[\frac{1}{2}(a + bx)]] - \frac{1}{(\cos[\frac{1}{2}(a + bx)] - \sin[\frac{1}{2}(a + bx)])^2} + \\ & \left. \frac{1}{(\cos[\frac{1}{2}(a + bx)] + \sin[\frac{1}{2}(a + bx)])^2} + 2 \tan[\frac{1}{2}(a + bx)] \right) \end{aligned}$$

Problem 11: Result more than twice size of optimal antiderivative.

$$\int \csc[2a + 2bx]^4 \sin[a + bx] dx$$

Optimal (type 3, 66 leaves, 6 steps):

$$-\frac{5 \operatorname{ArcTanh}[\cos[a+b x]]}{32 b} + \frac{5 \sec[a+b x]}{32 b} + \frac{5 \sec[a+b x]^3}{96 b} - \frac{\csc[a+b x]^2 \sec[a+b x]^3}{32 b}$$

Result (type 3, 205 leaves) :

$$\begin{aligned} & \frac{1}{24 b \left(\csc\left[\frac{1}{2} (a+b x)\right]^2 - \sec\left[\frac{1}{2} (a+b x)\right]^2\right)^3} \csc[a+b x]^8 \\ & \left(22 - 40 \cos[2 (a+b x)] + 13 \cos[3 (a+b x)] - 30 \cos[4 (a+b x)] + 13 \cos[5 (a+b x)] + \right. \\ & 15 \cos[3 (a+b x)] \log[\cos[\frac{1}{2} (a+b x)]] + 15 \cos[5 (a+b x)] \log[\cos[\frac{1}{2} (a+b x)]] - \\ & 15 \cos[3 (a+b x)] \log[\sin[\frac{1}{2} (a+b x)]] - 15 \cos[5 (a+b x)] \log[\sin[\frac{1}{2} (a+b x)]] + \\ & \left. \cos[a+b x] \left(-26 - 30 \log[\cos[\frac{1}{2} (a+b x)]] + 30 \log[\sin[\frac{1}{2} (a+b x)]]\right)\right) \end{aligned}$$

Problem 12: Result more than twice size of optimal antiderivative.

$$\int \csc[2 a + 2 b x]^5 \sin[a+b x] dx$$

Optimal (type 3, 89 leaves, 7 steps) :

$$\begin{aligned} & \frac{35 \operatorname{ArcTanh}[\sin[a+b x]]}{256 b} - \frac{35 \csc[a+b x]}{256 b} - \frac{35 \csc[a+b x]^3}{768 b} + \\ & \frac{7 \csc[a+b x]^3 \sec[a+b x]^2}{256 b} + \frac{\csc[a+b x]^3 \sec[a+b x]^4}{128 b} \end{aligned}$$

Result (type 3, 277 leaves) :

$$\begin{aligned} & -\frac{19 \cot[\frac{1}{2} (a+b x)]}{384 b} - \frac{\cot[\frac{1}{2} (a+b x)] \csc[\frac{1}{2} (a+b x)]^2}{768 b} - \\ & \frac{35 \log[\cos[\frac{1}{2} (a+b x)] - \sin[\frac{1}{2} (a+b x)]]}{256 b} + \frac{35 \log[\cos[\frac{1}{2} (a+b x)] + \sin[\frac{1}{2} (a+b x)]]}{256 b} + \\ & \frac{1}{512 b \left(\cos[\frac{1}{2} (a+b x)] - \sin[\frac{1}{2} (a+b x)]\right)^4} + \frac{11}{512 b \left(\cos[\frac{1}{2} (a+b x)] - \sin[\frac{1}{2} (a+b x)]\right)^2} - \\ & \frac{1}{512 b \left(\cos[\frac{1}{2} (a+b x)] + \sin[\frac{1}{2} (a+b x)]\right)^4} - \frac{11}{512 b \left(\cos[\frac{1}{2} (a+b x)] + \sin[\frac{1}{2} (a+b x)]\right)^2} - \\ & \frac{19 \tan[\frac{1}{2} (a+b x)]}{384 b} - \frac{\sec[\frac{1}{2} (a+b x)]^2 \tan[\frac{1}{2} (a+b x)]}{768 b} \end{aligned}$$

Problem 28: Result more than twice size of optimal antiderivative.

$$\int \csc[2 a + 2 b x] \sin[a+b x]^3 dx$$

Optimal (type 3, 28 leaves, 4 steps) :

$$\frac{\text{ArcTanh}[\sin[a + bx]]}{2b} - \frac{\sin[a + bx]}{2b}$$

Result (type 3, 71 leaves):

$$\begin{aligned} & \frac{1}{2} \left(-\frac{\log[\cos[\frac{1}{2}(a+bx)] - \sin[\frac{1}{2}(a+bx)]]}{b} + \right. \\ & \left. \frac{\log[\cos[\frac{1}{2}(a+bx)] + \sin[\frac{1}{2}(a+bx)]]}{b} - \frac{\sin[a+bx]}{b} \right) \end{aligned}$$

Problem 30: Result more than twice size of optimal antiderivative.

$$\int \csc[2a + 2bx]^3 \sin[a + bx]^3 dx$$

Optimal (type 3, 34 leaves, 3 steps):

$$\frac{\text{ArcTanh}[\sin[a + bx]]}{16b} + \frac{\sec[a + bx] \tan[a + bx]}{16b}$$

Result (type 3, 69 leaves):

$$\begin{aligned} & \frac{1}{16b} \left(-\log[\cos[\frac{1}{2}(a+bx)] - \sin[\frac{1}{2}(a+bx)]] + \right. \\ & \left. \log[\cos[\frac{1}{2}(a+bx)] + \sin[\frac{1}{2}(a+bx)]] + \sec[a+bx] \tan[a+bx] \right) \end{aligned}$$

Problem 32: Result more than twice size of optimal antiderivative.

$$\int \csc[2a + 2bx]^5 \sin[a + bx]^3 dx$$

Optimal (type 3, 70 leaves, 6 steps):

$$\frac{15 \text{ArcTanh}[\sin[a + bx]]}{256b} - \frac{15 \csc[a + bx]}{256b} + \frac{5 \csc[a + bx] \sec[a + bx]^2}{256b} + \frac{\csc[a + bx] \sec[a + bx]^4}{128b}$$

Result (type 3, 219 leaves):

$$\begin{aligned} & -\frac{\cot[\frac{1}{2}(a+bx)]}{64b} - \frac{15 \log[\cos[\frac{1}{2}(a+bx)] - \sin[\frac{1}{2}(a+bx)]]}{256b} + \\ & \frac{15 \log[\cos[\frac{1}{2}(a+bx)] + \sin[\frac{1}{2}(a+bx)]]}{256b} + \frac{1}{512b (\cos[\frac{1}{2}(a+bx)] - \sin[\frac{1}{2}(a+bx)])^4} - \\ & \frac{7}{512b (\cos[\frac{1}{2}(a+bx)] - \sin[\frac{1}{2}(a+bx)])^2} - \frac{1}{512b (\cos[\frac{1}{2}(a+bx)] + \sin[\frac{1}{2}(a+bx)])^4} - \\ & \frac{7}{512b (\cos[\frac{1}{2}(a+bx)] + \sin[\frac{1}{2}(a+bx)])^2} - \frac{\tan[\frac{1}{2}(a+bx)]}{64b} \end{aligned}$$

Problem 40: Result more than twice size of optimal antiderivative.

$$\int \csc[a + bx] \sin[2a + 2bx] dx$$

Optimal (type 3, 11 leaves, 2 steps):

$$\frac{2 \sin[a + bx]}{b}$$

Result (type 3, 23 leaves):

$$2 \left(\frac{\cos[bx] \sin[a]}{b} + \frac{\cos[a] \sin[bx]}{b} \right)$$

Problem 41: Result more than twice size of optimal antiderivative.

$$\int \csc[a + bx] \csc[2a + 2bx] dx$$

Optimal (type 3, 28 leaves, 4 steps):

$$\frac{\operatorname{ArcTanh}[\sin[a + bx]]}{2b} - \frac{\csc[a + bx]}{2b}$$

Result (type 3, 95 leaves):

$$-\frac{\cot\left[\frac{1}{2}(a + bx)\right]}{4b} - \frac{\log[\cos\left[\frac{1}{2}(a + bx)\right] - \sin\left[\frac{1}{2}(a + bx)\right]]}{2b} + \\ \frac{\log[\cos\left[\frac{1}{2}(a + bx)\right] + \sin\left[\frac{1}{2}(a + bx)\right]]}{2b} - \frac{\tan\left[\frac{1}{2}(a + bx)\right]}{4b}$$

Problem 42: Result more than twice size of optimal antiderivative.

$$\int \csc[a + bx] \csc[2a + 2bx]^2 dx$$

Optimal (type 3, 49 leaves, 5 steps):

$$-\frac{3 \operatorname{ArcTanh}[\cos[a + bx]]}{8b} + \frac{3 \sec[a + bx]}{8b} - \frac{\csc[a + bx]^2 \sec[a + bx]}{8b}$$

Result (type 3, 143 leaves):

$$\left(\csc[a + bx]^4 \left(2 - 6 \cos[2(a + bx)] + 2 \cos[3(a + bx)] + \right. \right. \\ \left. \left. 3 \cos[3(a + bx)] \log[\cos\left(\frac{1}{2}(a + bx)\right)] - 3 \cos[3(a + bx)] \log[\sin\left(\frac{1}{2}(a + bx)\right)] + \right. \right. \\ \left. \left. \cos[a + bx] \left(-2 - 3 \log[\cos\left(\frac{1}{2}(a + bx)\right)] + 3 \log[\sin\left(\frac{1}{2}(a + bx)\right)] \right) \right) \right) / \\ \left(8b \left(\csc\left(\frac{1}{2}(a + bx)\right)^2 - \sec\left(\frac{1}{2}(a + bx)\right)^2 \right) \right)$$

Problem 43: Result more than twice size of optimal antiderivative.

$$\int \csc[a + bx] \csc[2a + 2bx]^3 dx$$

Optimal (type 3, 66 leaves, 6 steps):

$$\frac{5 \operatorname{ArcTanh}[\sin[a + bx]]}{16 b} - \frac{5 \csc[a + bx]}{16 b} - \frac{5 \csc[a + bx]^3}{48 b} + \frac{\csc[a + bx]^3 \sec[a + bx]^2}{16 b}$$

Result (type 3, 215 leaves):

$$\begin{aligned} & -\frac{13 \cot\left[\frac{1}{2}(a + bx)\right]}{96 b} - \frac{\cot\left[\frac{1}{2}(a + bx)\right] \csc\left[\frac{1}{2}(a + bx)\right]^2}{192 b} - \\ & \frac{5 \log[\cos\left[\frac{1}{2}(a + bx)\right] - \sin\left[\frac{1}{2}(a + bx)\right]]}{16 b} + \frac{5 \log[\cos\left[\frac{1}{2}(a + bx)\right] + \sin\left[\frac{1}{2}(a + bx)\right]]}{16 b} + \\ & \frac{1}{32 b \left(\cos\left[\frac{1}{2}(a + bx)\right] - \sin\left[\frac{1}{2}(a + bx)\right]\right)^2} - \frac{32 b \left(\cos\left[\frac{1}{2}(a + bx)\right] + \sin\left[\frac{1}{2}(a + bx)\right]\right)^2}{192 b} - \\ & \frac{13 \tan\left[\frac{1}{2}(a + bx)\right]}{96 b} - \frac{\sec\left[\frac{1}{2}(a + bx)\right]^2 \tan\left[\frac{1}{2}(a + bx)\right]}{192 b} \end{aligned}$$

Problem 44: Result more than twice size of optimal antiderivative.

$$\int \csc[a + bx] \csc[2a + 2bx]^4 dx$$

Optimal (type 3, 89 leaves, 7 steps):

$$\begin{aligned} & -\frac{35 \operatorname{ArcTanh}[\cos[a + bx]]}{128 b} + \frac{35 \sec[a + bx]}{128 b} + \\ & \frac{35 \sec[a + bx]^3}{384 b} - \frac{7 \csc[a + bx]^2 \sec[a + bx]^3}{128 b} - \frac{\csc[a + bx]^4 \sec[a + bx]^3}{64 b} \end{aligned}$$

Result (type 3, 268 leaves):

$$\begin{aligned}
& - \frac{1}{384 b \left(\csc^2 \left[\frac{1}{2} (a + b x) \right]^2 - \sec^2 \left[\frac{1}{2} (a + b x) \right]^2 \right)^3} \csc^{10} [a + b x] \\
& \left(-204 + 658 \cos [2 (a + b x)] - 228 \cos [3 (a + b x)] + 140 \cos [4 (a + b x)] - 76 \cos [5 (a + b x)] - \right. \\
& 210 \cos [6 (a + b x)] + 76 \cos [7 (a + b x)] - 315 \cos [3 (a + b x)] \log [\cos [\frac{1}{2} (a + b x)]] - \\
& 105 \cos [5 (a + b x)] \log [\cos [\frac{1}{2} (a + b x)]] + 105 \cos [7 (a + b x)] \log [\cos [\frac{1}{2} (a + b x)]] + \\
& 3 \cos [a + b x] \left(76 + 105 \log [\cos [\frac{1}{2} (a + b x)]] - 105 \log [\sin [\frac{1}{2} (a + b x)]] \right) + \\
& 315 \cos [3 (a + b x)] \log [\sin [\frac{1}{2} (a + b x)]] + \\
& \left. 105 \cos [5 (a + b x)] \log [\sin [\frac{1}{2} (a + b x)]] - 105 \cos [7 (a + b x)] \log [\sin [\frac{1}{2} (a + b x)]] \right)
\end{aligned}$$

Problem 46: Result more than twice size of optimal antiderivative.

$$\int \csc^2 [a + b x] \sin [2 a + 2 b x]^7 dx$$

Optimal (type 3, 44 leaves, 5 steps):

$$-\frac{16 \cos [a + b x]^8}{b} + \frac{128 \cos [a + b x]^{10}}{5 b} - \frac{32 \cos [a + b x]^{12}}{3 b}$$

Result (type 3, 91 leaves):

$$\begin{aligned}
& -\frac{5 \cos [2 (a + b x)]}{8 b} - \frac{5 \cos [4 (a + b x)]}{64 b} + \frac{5 \cos [6 (a + b x)]}{48 b} + \\
& \frac{\cos [8 (a + b x)]}{32 b} - \frac{\cos [10 (a + b x)]}{80 b} - \frac{\cos [12 (a + b x)]}{192 b}
\end{aligned}$$

Problem 61: Result more than twice size of optimal antiderivative.

$$\int \csc^3 [a + b x] \sin [2 a + 2 b x]^8 dx$$

Optimal (type 3, 46 leaves, 4 steps):

$$-\frac{256 \cos [a + b x]^9}{9 b} + \frac{512 \cos [a + b x]^{11}}{11 b} - \frac{256 \cos [a + b x]^{13}}{13 b}$$

Result (type 3, 104 leaves):

$$\begin{aligned}
& -\frac{5 \cos [a + b x]}{4 b} - \frac{25 \cos [3 (a + b x)]}{48 b} + \frac{\cos [5 (a + b x)]}{16 b} + \\
& \frac{\cos [7 (a + b x)]}{8 b} + \frac{\cos [9 (a + b x)]}{72 b} - \frac{3 \cos [11 (a + b x)]}{176 b} - \frac{\cos [13 (a + b x)]}{208 b}
\end{aligned}$$

Problem 69: Result more than twice size of optimal antiderivative.

$$\int \csc[a + bx]^3 \csc[2a + 2bx] dx$$

Optimal (type 3, 43 leaves, 5 steps):

$$\frac{\operatorname{ArcTanh}[\sin[a + bx]]}{2b} - \frac{\csc[a + bx]}{2b} - \frac{\csc[a + bx]^3}{6b}$$

Result (type 3, 153 leaves):

$$\begin{aligned} & -\frac{7 \cot[\frac{1}{2}(a + bx)]}{24b} - \frac{\cot[\frac{1}{2}(a + bx)] \csc[\frac{1}{2}(a + bx)]^2}{48b} - \\ & \frac{\log[\cos[\frac{1}{2}(a + bx)] - \sin[\frac{1}{2}(a + bx)]]}{2b} + \frac{\log[\cos[\frac{1}{2}(a + bx)] + \sin[\frac{1}{2}(a + bx)]]}{2b} - \\ & \frac{7 \tan[\frac{1}{2}(a + bx)]}{24b} - \frac{\sec[\frac{1}{2}(a + bx)]^2 \tan[\frac{1}{2}(a + bx)]}{48b} \end{aligned}$$

Problem 70: Result more than twice size of optimal antiderivative.

$$\int \csc[a + bx]^3 \csc[2a + 2bx]^2 dx$$

Optimal (type 3, 70 leaves, 6 steps):

$$\begin{aligned} & -\frac{15 \operatorname{ArcTanh}[\cos[a + bx]]}{32b} + \frac{15 \sec[a + bx]}{32b} - \\ & \frac{5 \csc[a + bx]^2 \sec[a + bx]}{32b} - \frac{\csc[a + bx]^4 \sec[a + bx]}{16b} \end{aligned}$$

Result (type 3, 195 leaves):

$$\begin{aligned} & -\frac{7 \csc[\frac{1}{2}(a + bx)]^2}{128b} - \frac{\csc[\frac{1}{2}(a + bx)]^4}{256b} - \frac{15 \log[\cos[\frac{1}{2}(a + bx)]]}{32b} + \\ & \frac{15 \log[\sin[\frac{1}{2}(a + bx)]]}{32b} + \frac{7 \sec[\frac{1}{2}(a + bx)]^2}{128b} + \frac{\sec[\frac{1}{2}(a + bx)]^4}{256b} + \\ & \frac{\sin[\frac{1}{2}(a + bx)]}{4b (\cos[\frac{1}{2}(a + bx)] - \sin[\frac{1}{2}(a + bx)])} - \frac{\sin[\frac{1}{2}(a + bx)]}{4b (\cos[\frac{1}{2}(a + bx)] + \sin[\frac{1}{2}(a + bx)])} \end{aligned}$$

Problem 71: Result more than twice size of optimal antiderivative.

$$\int \csc[a + bx]^3 \csc[2a + 2bx]^3 dx$$

Optimal (type 3, 81 leaves, 6 steps):

$$\frac{7 \operatorname{ArcTanh}[\sin[a+b x]]}{16 b} - \frac{7 \csc[a+b x]}{48 b^3} - \frac{16 b}{7 \csc[a+b x]^5} + \frac{16 b}{80 b} + \frac{\csc[a+b x]^5 \sec[a+b x]^2}{16 b}$$

Result (type 3, 222 leaves) :

$$\begin{aligned} & -\frac{1}{3840 b} \left(818 \cot\left[\frac{1}{2} (a+b x)\right] + 1680 \log\left[\cos\left[\frac{1}{2} (a+b x)\right]\right] - \sin\left[\frac{1}{2} (a+b x)\right] \right. \\ & \quad - \frac{1680 \log\left[\cos\left[\frac{1}{2} (a+b x)\right]\right] + \sin\left[\frac{1}{2} (a+b x)\right]}{\left(\cos\left[\frac{1}{2} (a+b x)\right] - \sin\left[\frac{1}{2} (a+b x)\right]\right)^2} + \\ & \quad 392 \csc[a+b x]^3 \sin\left[\frac{1}{2} (a+b x)\right]^4 + 96 \csc[a+b x]^5 \sin\left[\frac{1}{2} (a+b x)\right]^6 + \\ & \quad \frac{120}{\left(\cos\left[\frac{1}{2} (a+b x)\right] + \sin\left[\frac{1}{2} (a+b x)\right]\right)^2} + \frac{49}{2} \csc\left[\frac{1}{2} (a+b x)\right]^4 \sin[a+b x] + \\ & \quad \left. \frac{3}{2} \csc\left[\frac{1}{2} (a+b x)\right]^6 \sin[a+b x] + 818 \tan\left[\frac{1}{2} (a+b x)\right] \right) \end{aligned}$$

Problem 72: Result more than twice size of optimal antiderivative.

$$\int \csc[a+b x]^3 \csc[2 a+2 b x]^4 dx$$

Optimal (type 3, 112 leaves, 8 steps) :

$$\begin{aligned} & -\frac{105 \operatorname{ArcTanh}[\cos[a+b x]]}{256 b} + \frac{105 \sec[a+b x]}{256 b} + \frac{35 \sec[a+b x]^3}{256 b} - \\ & \frac{21 \csc[a+b x]^2 \sec[a+b x]^3}{256 b} - \frac{3 \csc[a+b x]^4 \sec[a+b x]^3}{128 b} - \frac{\csc[a+b x]^6 \sec[a+b x]^3}{96 b} \end{aligned}$$

Result (type 3, 278 leaves) :

$$\frac{1}{3072 b \left(\csc\left[\frac{1}{2} (a+b x)\right]^2 - \sec\left[\frac{1}{2} (a+b x)\right]^2\right)^3}$$

$$\csc[a+b x]^{12} \left(1150 - 4752 \cos[2 (a+b x)] + 1600 \cos[3 (a+b x)] + 504 \cos[4 (a+b x)] + 1680 \cos[6 (a+b x)] - 600 \cos[7 (a+b x)] - 630 \cos[8 (a+b x)] + 200 \cos[9 (a+b x)] + 2520 \cos[3 (a+b x)] \log[\cos[\frac{1}{2} (a+b x)]] - 945 \cos[7 (a+b x)] \log[\cos[\frac{1}{2} (a+b x)]] + 315 \cos[9 (a+b x)] \log[\cos[\frac{1}{2} (a+b x)]] - 30 \cos[a+b x] \left(40 + 63 \log[\cos[\frac{1}{2} (a+b x)]] - 63 \log[\sin[\frac{1}{2} (a+b x)]]\right) - 2520 \cos[3 (a+b x)] \log[\sin[\frac{1}{2} (a+b x)]] + 945 \cos[7 (a+b x)] \log[\sin[\frac{1}{2} (a+b x)]] - 315 \cos[9 (a+b x)] \log[\sin[\frac{1}{2} (a+b x)]]\right)$$

Problem 123: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \sin[a+b x]^3 \sin[2 a+2 b x]^m dx$$

Optimal (type 5, 84 leaves, 2 steps):

$$\frac{1}{b (4+m)} (\cos[a+b x]^2)^{\frac{1-m}{2}} \text{Hypergeometric2F1}\left[\frac{1-m}{2}, \frac{4+m}{2}, \frac{6+m}{2}, \sin[a+b x]^2\right]$$

$$\sin[a+b x]^3 \sin[2 a+2 b x]^m \tan[a+b x]$$

Result (type 6, 5212 leaves):

$$\begin{aligned} & \left(2^{4+m} (4+m) \cos\left[\frac{1}{2} (a+b x)\right]^6 \sin\left[\frac{1}{2} (a+b x)\right]^2 \right. \\ & \left. \sin[a+b x]^3 \left(\cos\left[\frac{1}{2} (a+b x)\right] \left(-\sin\left[\frac{1}{2} (a+b x)\right] + \sin\left[\frac{3}{2} (a+b x)\right]\right)\right)^m \right. \\ & \left. \sin[2 (a+b x)]^m \left(\left(\text{AppellF1}\left[\frac{2+m}{2}, -m, 3+2m, \frac{4+m}{2}, \right.\right.\right. \right. \\ & \left. \left. \left. \left. \tan\left[\frac{1}{2} (a+b x)\right]^2, -\tan\left[\frac{1}{2} (a+b x)\right]^2\right] \sec\left[\frac{1}{2} (a+b x)\right]^2\right)\right. \right. \\ & \left. \left. \left. \left. \left((4+m) \text{AppellF1}\left[\frac{2+m}{2}, -m, 3+2m, \frac{4+m}{2}, \tan\left[\frac{1}{2} (a+b x)\right]^2, -\tan\left[\frac{1}{2} (a+b x)\right]^2\right] - \right.\right.\right. \right. \\ & \left. \left. \left. \left. 2 \left(m \text{AppellF1}\left[\frac{4+m}{2}, 1-m, 3+2m, \frac{6+m}{2}, \tan\left[\frac{1}{2} (a+b x)\right]^2, -\tan\left[\frac{1}{2} (a+b x)\right]^2\right] + \right.\right.\right. \right. \\ & \left. \left. \left. \left. (3+2m) \text{AppellF1}\left[\frac{4+m}{2}, -m, 2(2+m), \frac{6+m}{2}, \right.\right.\right. \right. \\ & \left. \left. \left. \left. \tan\left[\frac{1}{2} (a+b x)\right]^2, -\tan\left[\frac{1}{2} (a+b x)\right]^2\right] \right) \tan\left[\frac{1}{2} (a+b x)\right]^2\right) - \right. \\ & \left. \left. \left. \left. \text{AppellF1}\left[\frac{2+m}{2}, -m, 2(2+m), \frac{4+m}{2}, \tan\left[\frac{1}{2} (a+b x)\right]^2, -\tan\left[\frac{1}{2} (a+b x)\right]^2\right]\right)\right)\right) \end{aligned}$$

$$\begin{aligned}
& \left((4+m) \operatorname{AppellF1}\left[\frac{2+m}{2}, -m, 2(2+m), \frac{4+m}{2}, \tan\left[\frac{1}{2}(a+b x)\right]^2, -\tan\left[\frac{1}{2}(a+b x)\right]^2\right] - \right. \\
& 2 \left(m \operatorname{AppellF1}\left[\frac{4+m}{2}, 1-m, 2(2+m), \frac{6+m}{2}, \tan\left[\frac{1}{2}(a+b x)\right]^2, -\tan\left[\frac{1}{2}(a+b x)\right]^2\right] + \right. \\
& 2(2+m) \operatorname{AppellF1}\left[\frac{4+m}{2}, -m, 5+2m, \frac{6+m}{2}, \tan\left[\frac{1}{2}(a+b x)\right]^2, \right. \\
& \left. \left. -\tan\left[\frac{1}{2}(a+b x)\right]^2\right] \tan\left[\frac{1}{2}(a+b x)\right]^2\right) \Bigg) / \\
& \left(b(2+m) \left(\frac{1}{2+m} 2^{4+m} (4+m) \cos\left[\frac{1}{2}(a+b x)\right]^7 \sin\left[\frac{1}{2}(a+b x)\right] \right. \right. \\
& \left. \left. \left(\cos\left[\frac{1}{2}(a+b x)\right] \left(-\sin\left[\frac{1}{2}(a+b x)\right] + \sin\left[\frac{3}{2}(a+b x)\right] \right) \right)^m \right. \\
& \left(\left(\operatorname{AppellF1}\left[\frac{2+m}{2}, -m, 3+2m, \frac{4+m}{2}, \tan\left[\frac{1}{2}(a+b x)\right]^2, -\tan\left[\frac{1}{2}(a+b x)\right]^2\right] \right. \right. \\
& \sec\left[\frac{1}{2}(a+b x)\right]^2 \Bigg) / \left((4+m) \operatorname{AppellF1}\left[\frac{2+m}{2}, -m, 3+2m, \frac{4+m}{2}, \tan\left[\frac{1}{2}(a+b x)\right]^2, \right. \right. \\
& \left. \left. -\tan\left[\frac{1}{2}(a+b x)\right]^2\right] - 2 \left(m \operatorname{AppellF1}\left[\frac{4+m}{2}, 1-m, 3+2m, \frac{6+m}{2}, \tan\left[\frac{1}{2}(a+b x)\right]^2, \right. \right. \\
& \left. \left. -\tan\left[\frac{1}{2}(a+b x)\right]^2\right] + (3+2m) \operatorname{AppellF1}\left[\frac{4+m}{2}, -m, 2(2+m), \right. \right. \\
& \left. \left. \frac{6+m}{2}, \tan\left[\frac{1}{2}(a+b x)\right]^2, -\tan\left[\frac{1}{2}(a+b x)\right]^2\right] \tan\left[\frac{1}{2}(a+b x)\right]^2 \right) - \\
& \operatorname{AppellF1}\left[\frac{2+m}{2}, -m, 2(2+m), \frac{4+m}{2}, \tan\left[\frac{1}{2}(a+b x)\right]^2, -\tan\left[\frac{1}{2}(a+b x)\right]^2\right] \Bigg) / \\
& \left((4+m) \operatorname{AppellF1}\left[\frac{2+m}{2}, -m, 2(2+m), \frac{4+m}{2}, \tan\left[\frac{1}{2}(a+b x)\right]^2, -\tan\left[\frac{1}{2}(a+b x)\right]^2\right] - \right. \\
& \left. 2 \left(m \operatorname{AppellF1}\left[\frac{4+m}{2}, 1-m, 2(2+m), \frac{6+m}{2}, \tan\left[\frac{1}{2}(a+b x)\right]^2, \right. \right. \right. \\
& \left. \left. \left. -\tan\left[\frac{1}{2}(a+b x)\right]^2\right] + 2(2+m) \operatorname{AppellF1}\left[\frac{4+m}{2}, -m, 5+2m, \frac{6+m}{2}, \right. \right. \\
& \left. \left. \tan\left[\frac{1}{2}(a+b x)\right]^2, -\tan\left[\frac{1}{2}(a+b x)\right]^2\right] \tan\left[\frac{1}{2}(a+b x)\right]^2 \right) - \right. \\
& \left. \frac{1}{2+m} 3 \times 2^{4+m} (4+m) \cos\left[\frac{1}{2}(a+b x)\right]^5 \sin\left[\frac{1}{2}(a+b x)\right]^3 \right. \\
& \left. \left(\cos\left[\frac{1}{2}(a+b x)\right] \left(-\sin\left[\frac{1}{2}(a+b x)\right] + \sin\left[\frac{3}{2}(a+b x)\right] \right) \right)^m \right. \\
& \left(\left(\operatorname{AppellF1}\left[\frac{2+m}{2}, -m, 3+2m, \frac{4+m}{2}, \tan\left[\frac{1}{2}(a+b x)\right]^2, -\tan\left[\frac{1}{2}(a+b x)\right]^2\right] \right. \right. \\
& \sec\left[\frac{1}{2}(a+b x)\right]^2 \Bigg) / \left((4+m) \operatorname{AppellF1}\left[\frac{2+m}{2}, -m, 3+2m, \frac{4+m}{2}, \tan\left[\frac{1}{2}(a+b x)\right]^2, \right. \right. \\
& \left. \left. -\tan\left[\frac{1}{2}(a+b x)\right]^2\right] - 2 \left(m \operatorname{AppellF1}\left[\frac{4+m}{2}, 1-m, 3+2m, \frac{6+m}{2}, \tan\left[\frac{1}{2}(a+b x)\right]^2, \right. \right. \\
& \left. \left. -\tan\left[\frac{1}{2}(a+b x)\right]^2\right] + (3+2m) \operatorname{AppellF1}\left[\frac{4+m}{2}, -m, 2(2+m), \right. \right. \\
& \left. \left. \frac{6+m}{2}, \tan\left[\frac{1}{2}(a+b x)\right]^2, -\tan\left[\frac{1}{2}(a+b x)\right]^2\right] \tan\left[\frac{1}{2}(a+b x)\right]^2 \right) -
\end{aligned}$$

$$\begin{aligned}
& \text{AppellF1}\left[\frac{2+m}{2}, -m, 2(2+m), \frac{4+m}{2}, \tan\left[\frac{1}{2}(a+b x)\right]^2, -\tan\left[\frac{1}{2}(a+b x)\right]^2\right] / \\
& \quad \left((4+m) \text{AppellF1}\left[\frac{2+m}{2}, -m, 2(2+m), \frac{4+m}{2}, \tan\left[\frac{1}{2}(a+b x)\right]^2, -\tan\left[\frac{1}{2}(a+b x)\right]^2\right] - \right. \\
& \quad \left. 2 \left(m \text{AppellF1}\left[\frac{4+m}{2}, 1-m, 2(2+m), \frac{6+m}{2}, \tan\left[\frac{1}{2}(a+b x)\right]^2, \right.\right. \right. \\
& \quad \left. \left. -\tan\left[\frac{1}{2}(a+b x)\right]^2\right] + 2(2+m) \text{AppellF1}\left[\frac{4+m}{2}, -m, 5+2m, \frac{6+m}{2}, \right. \\
& \quad \left. \left. \tan\left[\frac{1}{2}(a+b x)\right]^2, -\tan\left[\frac{1}{2}(a+b x)\right]^2\right]\right) \tan\left[\frac{1}{2}(a+b x)\right]^2\right] + \\
& \frac{1}{2+m} 2^{4+m} m (4+m) \cos\left[\frac{1}{2}(a+b x)\right]^6 \sin\left[\frac{1}{2}(a+b x)\right]^2 \\
& \quad \left(\cos\left[\frac{1}{2}(a+b x)\right] \left(-\sin\left[\frac{1}{2}(a+b x)\right] + \sin\left[\frac{3}{2}(a+b x)\right]\right)^{-1+m} \right. \\
& \quad \left(\cos\left[\frac{1}{2}(a+b x)\right] \left(-\frac{1}{2} \cos\left[\frac{1}{2}(a+b x)\right] + \frac{3}{2} \cos\left[\frac{3}{2}(a+b x)\right]\right) - \right. \\
& \quad \left. \frac{1}{2} \sin\left[\frac{1}{2}(a+b x)\right] \left(-\sin\left[\frac{1}{2}(a+b x)\right] + \sin\left[\frac{3}{2}(a+b x)\right]\right)\right) \\
& \quad \left(\left(\text{AppellF1}\left[\frac{2+m}{2}, -m, 3+2m, \frac{4+m}{2}, \tan\left[\frac{1}{2}(a+b x)\right]^2, -\tan\left[\frac{1}{2}(a+b x)\right]^2\right] \right. \right. \\
& \quad \left. \left. \sec\left[\frac{1}{2}(a+b x)\right]^2\right) / \left((4+m) \text{AppellF1}\left[\frac{2+m}{2}, -m, 3+2m, \frac{4+m}{2}, \tan\left[\frac{1}{2}(a+b x)\right]^2, \right. \right. \right. \\
& \quad \left. \left. -\tan\left[\frac{1}{2}(a+b x)\right]^2\right] - 2 \left(m \text{AppellF1}\left[\frac{4+m}{2}, 1-m, 3+2m, \frac{6+m}{2}, \tan\left[\frac{1}{2}(a+b x)\right]^2, \right. \right. \right. \\
& \quad \left. \left. -\tan\left[\frac{1}{2}(a+b x)\right]^2\right] + (3+2m) \text{AppellF1}\left[\frac{4+m}{2}, -m, 2(2+m), \right. \right. \\
& \quad \left. \left. \frac{6+m}{2}, \tan\left[\frac{1}{2}(a+b x)\right]^2, -\tan\left[\frac{1}{2}(a+b x)\right]^2\right]\right) \tan\left[\frac{1}{2}(a+b x)\right]^2\right) - \\
& \text{AppellF1}\left[\frac{2+m}{2}, -m, 2(2+m), \frac{4+m}{2}, \tan\left[\frac{1}{2}(a+b x)\right]^2, -\tan\left[\frac{1}{2}(a+b x)\right]^2\right] / \\
& \quad \left((4+m) \text{AppellF1}\left[\frac{2+m}{2}, -m, 2(2+m), \frac{4+m}{2}, \tan\left[\frac{1}{2}(a+b x)\right]^2, -\tan\left[\frac{1}{2}(a+b x)\right]^2\right] - \right. \\
& \quad \left. 2 \left(m \text{AppellF1}\left[\frac{4+m}{2}, 1-m, 2(2+m), \frac{6+m}{2}, \tan\left[\frac{1}{2}(a+b x)\right]^2, -\tan\left[\frac{1}{2}(a+b x)\right]^2\right] + \right. \right. \\
& \quad \left. \left. 2(2+m) \text{AppellF1}\left[\frac{4+m}{2}, -m, 5+2m, \frac{6+m}{2}, \tan\left[\frac{1}{2}(a+b x)\right]^2, \right. \right. \right. \\
& \quad \left. \left. -\tan\left[\frac{1}{2}(a+b x)\right]^2\right]\right) \tan\left[\frac{1}{2}(a+b x)\right]^2\right) + \frac{1}{2+m} 2^{4+m} (4+m) \cos\left[\frac{1}{2}(a+b x)\right]^6 \\
& \quad \sin\left[\frac{1}{2}(a+b x)\right]^2 \left(\cos\left[\frac{1}{2}(a+b x)\right] \left(-\sin\left[\frac{1}{2}(a+b x)\right] + \sin\left[\frac{3}{2}(a+b x)\right]\right)\right)^m \\
& \quad \left(\left(\text{AppellF1}\left[\frac{2+m}{2}, -m, 3+2m, \frac{4+m}{2}, \tan\left[\frac{1}{2}(a+b x)\right]^2, -\tan\left[\frac{1}{2}(a+b x)\right]^2\right] \right. \right. \\
& \quad \left. \left. \sec\left[\frac{1}{2}(a+b x)\right]^2 \tan\left[\frac{1}{2}(a+b x)\right]\right) / \right. \\
& \quad \left. \left((4+m) \text{AppellF1}\left[\frac{2+m}{2}, -m, 3+2m, \frac{4+m}{2}, \tan\left[\frac{1}{2}(a+b x)\right]^2, -\tan\left[\frac{1}{2}(a+b x)\right]^2\right] - \right. \right.
\end{aligned}$$

$$\begin{aligned}
& 2 \left(m \operatorname{AppellF1} \left[\frac{4+m}{2}, 1-m, 3+2m, \frac{6+m}{2}, \tan \left[\frac{1}{2} (a+b x) \right]^2, -\tan \left[\frac{1}{2} (a+b x) \right]^2 \right] + \right. \\
& \quad \left. (3+2m) \operatorname{AppellF1} \left[\frac{4+m}{2}, -m, 2(2+m), \frac{6+m}{2}, \right. \right. \\
& \quad \left. \left. \tan \left[\frac{1}{2} (a+b x) \right]^2, -\tan \left[\frac{1}{2} (a+b x) \right]^2 \right] \right) \tan \left[\frac{1}{2} (a+b x) \right]^2 + \\
& \left(\sec \left[\frac{1}{2} (a+b x) \right]^2 \left(-\frac{1}{4+m} m (2+m) \operatorname{AppellF1} \left[1+\frac{2+m}{2}, 1-m, 3+2m, 1+\frac{4+m}{2}, \right. \right. \right. \\
& \quad \left. \left. \tan \left[\frac{1}{2} (a+b x) \right]^2, -\tan \left[\frac{1}{2} (a+b x) \right]^2 \right] \sec \left[\frac{1}{2} (a+b x) \right]^2 \tan \left[\frac{1}{2} (a+b x) \right] - \right. \\
& \quad \left. \left. \frac{1}{4+m} (2+m) (3+2m) \operatorname{AppellF1} \left[1+\frac{2+m}{2}, -m, 4+2m, 1+\frac{4+m}{2}, \right. \right. \right. \\
& \quad \left. \left. \tan \left[\frac{1}{2} (a+b x) \right]^2, -\tan \left[\frac{1}{2} (a+b x) \right]^2 \right] \sec \left[\frac{1}{2} (a+b x) \right]^2 \tan \left[\frac{1}{2} (a+b x) \right] \right) \right) / \\
& \left((4+m) \operatorname{AppellF1} \left[\frac{2+m}{2}, -m, 3+2m, \frac{4+m}{2}, \tan \left[\frac{1}{2} (a+b x) \right]^2, -\tan \left[\frac{1}{2} (a+b x) \right]^2 \right] - \right. \\
& \quad \left. 2 \left(m \operatorname{AppellF1} \left[\frac{4+m}{2}, 1-m, 3+2m, \frac{6+m}{2}, \tan \left[\frac{1}{2} (a+b x) \right]^2, -\tan \left[\frac{1}{2} (a+b x) \right]^2 \right] + \right. \right. \\
& \quad \left. \left. (3+2m) \operatorname{AppellF1} \left[\frac{4+m}{2}, -m, 2(2+m), \frac{6+m}{2}, \right. \right. \right. \\
& \quad \left. \left. \tan \left[\frac{1}{2} (a+b x) \right]^2, -\tan \left[\frac{1}{2} (a+b x) \right]^2 \right] \right) \tan \left[\frac{1}{2} (a+b x) \right]^2 - \\
& \quad \left(-\frac{1}{4+m} m (2+m) \operatorname{AppellF1} \left[1+\frac{2+m}{2}, 1-m, 2(2+m), 1+\frac{4+m}{2}, \tan \left[\frac{1}{2} (a+b x) \right]^2, \right. \right. \\
& \quad \left. \left. -\tan \left[\frac{1}{2} (a+b x) \right]^2 \right] \sec \left[\frac{1}{2} (a+b x) \right]^2 \tan \left[\frac{1}{2} (a+b x) \right] - \frac{1}{4+m} \right. \\
& \quad \left. 2(2+m)^2 \operatorname{AppellF1} \left[1+\frac{2+m}{2}, -m, 1+2(2+m), 1+\frac{4+m}{2}, \tan \left[\frac{1}{2} (a+b x) \right]^2, \right. \right. \\
& \quad \left. \left. -\tan \left[\frac{1}{2} (a+b x) \right]^2 \right] \sec \left[\frac{1}{2} (a+b x) \right]^2 \tan \left[\frac{1}{2} (a+b x) \right] \right) / \\
& \left((4+m) \operatorname{AppellF1} \left[\frac{2+m}{2}, -m, 2(2+m), \frac{4+m}{2}, \tan \left[\frac{1}{2} (a+b x) \right]^2, -\tan \left[\frac{1}{2} (a+b x) \right]^2 \right] - \right. \\
& \quad \left. 2 \left(m \operatorname{AppellF1} \left[\frac{4+m}{2}, 1-m, 2(2+m), \frac{6+m}{2}, \tan \left[\frac{1}{2} (a+b x) \right]^2, \right. \right. \right. \\
& \quad \left. \left. -\tan \left[\frac{1}{2} (a+b x) \right]^2 \right] + 2(2+m) \operatorname{AppellF1} \left[\frac{4+m}{2}, -m, 5+2m, \frac{6+m}{2}, \right. \right. \\
& \quad \left. \left. \tan \left[\frac{1}{2} (a+b x) \right]^2, -\tan \left[\frac{1}{2} (a+b x) \right]^2 \right] \right) \tan \left[\frac{1}{2} (a+b x) \right]^2 + \\
& \quad \left(\operatorname{AppellF1} \left[\frac{2+m}{2}, -m, 2(2+m), \frac{4+m}{2}, \tan \left[\frac{1}{2} (a+b x) \right]^2, -\tan \left[\frac{1}{2} (a+b x) \right]^2 \right] \right. \\
& \quad \left. \left(-2 \left(m \operatorname{AppellF1} \left[\frac{4+m}{2}, 1-m, 2(2+m), \frac{6+m}{2}, \tan \left[\frac{1}{2} (a+b x) \right]^2, \right. \right. \right. \right. \\
& \quad \left. \left. -\tan \left[\frac{1}{2} (a+b x) \right]^2 \right] + 2(2+m) \operatorname{AppellF1} \left[\frac{4+m}{2}, -m, 5+2m, \frac{6+m}{2}, \right. \right. \\
& \quad \left. \left. \tan \left[\frac{1}{2} (a+b x) \right]^2, -\tan \left[\frac{1}{2} (a+b x) \right]^2 \right] \right) \sec \left[\frac{1}{2} (a+b x) \right]^2 \tan \left[\frac{1}{2} (a+b x) \right] + \right)
\end{aligned}$$

$$\begin{aligned}
& (4 + m) \left(-\frac{1}{4 + m} m (2 + m) \text{AppellF1}\left[1 + \frac{2 + m}{2}, 1 - m, 2 (2 + m), 1 + \frac{4 + m}{2}, \right. \right. \\
& \quad \left. \left. \tan\left[\frac{1}{2} (a + b x)\right]^2, -\tan\left[\frac{1}{2} (a + b x)\right]^2\right] \sec\left[\frac{1}{2} (a + b x)\right]^2 \tan\left[\frac{1}{2} (a + b x)\right] - \right. \\
& \quad \left. \frac{1}{4 + m} 2 (2 + m)^2 \text{AppellF1}\left[1 + \frac{2 + m}{2}, -m, 1 + 2 (2 + m), 1 + \frac{4 + m}{2}, \right. \right. \\
& \quad \left. \left. \tan\left[\frac{1}{2} (a + b x)\right]^2, -\tan\left[\frac{1}{2} (a + b x)\right]^2\right] \sec\left[\frac{1}{2} (a + b x)\right]^2 \tan\left[\frac{1}{2} (a + b x)\right] \right) - \\
& 2 \tan\left[\frac{1}{2} (a + b x)\right]^2 \left(m \left(-\frac{1}{6 + m} 2 (2 + m) (4 + m) \text{AppellF1}\left[1 + \frac{4 + m}{2}, 1 - m, \right. \right. \right. \\
& \quad \left. \left. 1 + 2 (2 + m), 1 + \frac{6 + m}{2}, \tan\left[\frac{1}{2} (a + b x)\right]^2, -\tan\left[\frac{1}{2} (a + b x)\right]^2\right] \sec\left[\frac{1}{2} \right. \right. \\
& \quad \left. \left. (a + b x)\right]^2 \tan\left[\frac{1}{2} (a + b x)\right] + \frac{1}{6 + m} (1 - m) (4 + m) \text{AppellF1}\left[1 + \frac{4 + m}{2}, \right. \right. \\
& \quad \left. \left. 2 - m, 2 (2 + m), 1 + \frac{6 + m}{2}, \tan\left[\frac{1}{2} (a + b x)\right]^2, -\tan\left[\frac{1}{2} (a + b x)\right]^2\right] \right. \\
& \quad \left. \sec\left[\frac{1}{2} (a + b x)\right]^2 \tan\left[\frac{1}{2} (a + b x)\right] \right) + 2 (2 + m) \left(-\frac{1}{6 + m} m (4 + m) \right. \\
& \quad \left. \text{AppellF1}\left[1 + \frac{4 + m}{2}, 1 - m, 5 + 2 m, 1 + \frac{6 + m}{2}, \tan\left[\frac{1}{2} (a + b x)\right]^2, \right. \right. \\
& \quad \left. \left. -\tan\left[\frac{1}{2} (a + b x)\right]^2\right] \sec\left[\frac{1}{2} (a + b x)\right]^2 \tan\left[\frac{1}{2} (a + b x)\right] - \frac{1}{6 + m} (4 + m) \right. \\
& \quad \left. (5 + 2 m) \text{AppellF1}\left[1 + \frac{4 + m}{2}, -m, 6 + 2 m, 1 + \frac{6 + m}{2}, \tan\left[\frac{1}{2} (a + b x)\right]^2, \right. \right. \\
& \quad \left. \left. -\tan\left[\frac{1}{2} (a + b x)\right]^2\right] \sec\left[\frac{1}{2} (a + b x)\right]^2 \tan\left[\frac{1}{2} (a + b x)\right] \right) \right) \Bigg) / \\
& \left((4 + m) \text{AppellF1}\left[\frac{2 + m}{2}, -m, 2 (2 + m), \frac{4 + m}{2}, \tan\left[\frac{1}{2} (a + b x)\right]^2, \right. \right. \\
& \quad \left. \left. -\tan\left[\frac{1}{2} (a + b x)\right]^2\right] - 2 \left(m \text{AppellF1}\left[\frac{4 + m}{2}, 1 - m, 2 (2 + m), \frac{6 + m}{2}, \right. \right. \right. \\
& \quad \left. \left. \tan\left[\frac{1}{2} (a + b x)\right]^2, -\tan\left[\frac{1}{2} (a + b x)\right]^2\right] + 2 (2 + m) \text{AppellF1}\left[\frac{4 + m}{2}, -m, \right. \right. \\
& \quad \left. \left. 5 + 2 m, \frac{6 + m}{2}, \tan\left[\frac{1}{2} (a + b x)\right]^2, -\tan\left[\frac{1}{2} (a + b x)\right]^2\right] \right) \tan\left[\frac{1}{2} (a + b x)\right]^2 - \right. \\
& \quad \left. \left(\text{AppellF1}\left[\frac{2 + m}{2}, -m, 3 + 2 m, \frac{4 + m}{2}, \tan\left[\frac{1}{2} (a + b x)\right]^2, -\tan\left[\frac{1}{2} (a + b x)\right]^2\right] \right. \right. \\
& \quad \left. \left. \sec\left[\frac{1}{2} (a + b x)\right]^2 \left(-2 \left(m \text{AppellF1}\left[\frac{4 + m}{2}, 1 - m, 3 + 2 m, \frac{6 + m}{2}, \tan\left[\frac{1}{2} (a + b x)\right]^2, \right. \right. \right. \right. \right. \right. \\
& \quad \left. \left. \left. \left. \left. -\tan\left[\frac{1}{2} (a + b x)\right]^2\right] + (3 + 2 m) \text{AppellF1}\left[\frac{4 + m}{2}, -m, 2 (2 + m), \frac{6 + m}{2}, \right. \right. \right. \right. \right. \right. \\
& \quad \left. \left. \left. \left. \left. \tan\left[\frac{1}{2} (a + b x)\right]^2, -\tan\left[\frac{1}{2} (a + b x)\right]^2\right] \right) \sec\left[\frac{1}{2} (a + b x)\right]^2 \tan\left[\frac{1}{2} (a + b x)\right] + \right. \right. \\
& \quad \left. \left. (4 + m) \left(-\frac{1}{4 + m} m (2 + m) \text{AppellF1}\left[1 + \frac{2 + m}{2}, 1 - m, 3 + 2 m, 1 + \frac{4 + m}{2}, \right. \right. \right. \right. \\
& \quad \left. \left. \tan\left[\frac{1}{2} (a + b x)\right]^2, -\tan\left[\frac{1}{2} (a + b x)\right]^2\right] \sec\left[\frac{1}{2} (a + b x)\right]^2 \tan\left[\frac{1}{2} (a + b x)\right] \right) - \right.
\end{aligned}$$

Problem 124: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \sin[a + bx]^2 \sin[2a + 2bx]^m dx$$

Optimal (type 5, 84 leaves, 2 steps):

$$\frac{1}{b(3+m)} \left(\cos[a+bx]^2\right)^{\frac{1-m}{2}} \text{Hypergeometric2F1}\left[\frac{1-m}{2}, \frac{3+m}{2}, \frac{5+m}{2}, \sin[a+bx]^2\right] \\ \sin[a+bx]^2 \sin[2a+2bx]^m \tan[a+bx]$$

Result (type 6, 5195 leaves):

$$\left(2^{3+m} (3+m) \cos\left[\frac{1}{2} (a+b x)\right]^5 \sin\left[\frac{1}{2} (a+b x)\right] \sin[a+b x]^2 \right. \\ \left. \left(\cos\left[\frac{1}{2} (a+b x)\right] \left(-\sin\left[\frac{1}{2} (a+b x)\right] + \sin\left[\frac{3}{2} (a+b x)\right] \right) \right)^m \sin[2 (a+b x)]^m \right)$$

$$\begin{aligned}
& \left(- \left(\text{AppellF1} \left[\frac{1+m}{2}, -m, 3+2m, \frac{3+m}{2}, \tan \left[\frac{1}{2} (a+b x) \right]^2, -\tan \left[\frac{1}{2} (a+b x) \right]^2 \right] \right. \right. \\
& \quad \left((3+m) \text{AppellF1} \left[\frac{1+m}{2}, -m, 3+2m, \frac{3+m}{2}, \tan \left[\frac{1}{2} (a+b x) \right]^2, -\tan \left[\frac{1}{2} (a+b x) \right]^2 \right] - \right. \\
& \quad \left. 2 \left(m \text{AppellF1} \left[\frac{3+m}{2}, 1-m, 3+2m, \frac{5+m}{2}, \tan \left[\frac{1}{2} (a+b x) \right]^2, -\tan \left[\frac{1}{2} (a+b x) \right]^2 \right] + \right. \right. \\
& \quad \left. (3+2m) \text{AppellF1} \left[\frac{3+m}{2}, -m, 2(2+m), \frac{5+m}{2}, \right. \right. \\
& \quad \left. \tan \left[\frac{1}{2} (a+b x) \right]^2, -\tan \left[\frac{1}{2} (a+b x) \right]^2 \right] \left. \right) \tan \left[\frac{1}{2} (a+b x) \right]^2 \right) + \\
& \left(\text{AppellF1} \left[\frac{1+m}{2}, -m, 2(1+m), \frac{3+m}{2}, \tan \left[\frac{1}{2} (a+b x) \right]^2, -\tan \left[\frac{1}{2} (a+b x) \right]^2 \right] \right. \\
& \quad \left. \sec \left[\frac{1}{2} (a+b x) \right]^2 \right) \left/ \right. \\
& \quad \left((3+m) \text{AppellF1} \left[\frac{1+m}{2}, -m, 2(1+m), \frac{3+m}{2}, \tan \left[\frac{1}{2} (a+b x) \right]^2, -\tan \left[\frac{1}{2} (a+b x) \right]^2 \right] - \right. \\
& \quad \left. 2 \left(m \text{AppellF1} \left[\frac{3+m}{2}, 1-m, 2(1+m), \frac{5+m}{2}, \tan \left[\frac{1}{2} (a+b x) \right]^2, -\tan \left[\frac{1}{2} (a+b x) \right]^2 \right] + \right. \right. \\
& \quad \left. 2(1+m) \text{AppellF1} \left[\frac{3+m}{2}, -m, 3+2m, \frac{5+m}{2}, \tan \left[\frac{1}{2} (a+b x) \right]^2, -\tan \left[\frac{1}{2} (a+b x) \right]^2 \right] \right) \\
& \quad \left. \tan \left[\frac{1}{2} (a+b x) \right]^2 \right) \left/ \right. \left(b(1+m) \right. \\
& \quad \left. \left(\frac{1}{1+m} 2^{2+m} (3+m) \cos \left[\frac{1}{2} (a+b x) \right]^6 \left(\cos \left[\frac{1}{2} (a+b x) \right] \left(-\sin \left[\frac{1}{2} (a+b x) \right] + \sin \left[\frac{3}{2} (a+b x) \right] \right) \right)^m \right. \right. \\
& \quad \left. \left(- \left(\text{AppellF1} \left[\frac{1+m}{2}, -m, 3+2m, \frac{3+m}{2}, \tan \left[\frac{1}{2} (a+b x) \right]^2, -\tan \left[\frac{1}{2} (a+b x) \right]^2 \right] \right. \right. \right. \\
& \quad \left. \left((3+m) \text{AppellF1} \left[\frac{1+m}{2}, -m, 3+2m, \frac{3+m}{2}, \tan \left[\frac{1}{2} (a+b x) \right]^2, -\tan \left[\frac{1}{2} (a+b x) \right]^2 \right] - \right. \right. \\
& \quad \left. \left. 2 \left(m \text{AppellF1} \left[\frac{3+m}{2}, 1-m, 3+2m, \frac{5+m}{2}, \tan \left[\frac{1}{2} (a+b x) \right]^2, -\tan \left[\frac{1}{2} (a+b x) \right]^2 \right] + \right. \right. \right. \\
& \quad \left. (3+2m) \text{AppellF1} \left[\frac{3+m}{2}, -m, 2(2+m), \frac{5+m}{2}, \right. \right. \\
& \quad \left. \tan \left[\frac{1}{2} (a+b x) \right]^2, -\tan \left[\frac{1}{2} (a+b x) \right]^2 \right] \tan \left[\frac{1}{2} (a+b x) \right]^2 \right) \right. \\
& \quad \left. \left(\text{AppellF1} \left[\frac{1+m}{2}, -m, 2(1+m), \frac{3+m}{2}, \tan \left[\frac{1}{2} (a+b x) \right]^2, -\tan \left[\frac{1}{2} (a+b x) \right]^2 \right] \right. \right. \\
& \quad \left. \sec \left[\frac{1}{2} (a+b x) \right]^2 \right) \left/ \right. \left((3+m) \text{AppellF1} \left[\frac{1+m}{2}, -m, 2(1+m), \frac{3+m}{2}, \right. \right. \\
& \quad \left. \tan \left[\frac{1}{2} (a+b x) \right]^2, -\tan \left[\frac{1}{2} (a+b x) \right]^2 \right] - 2 \left(m \text{AppellF1} \left[\frac{3+m}{2}, 1-m, 2(1+m), \right. \right. \\
& \quad \left. \frac{5+m}{2}, \tan \left[\frac{1}{2} (a+b x) \right]^2, -\tan \left[\frac{1}{2} (a+b x) \right]^2 \right] + 2(1+m) \text{AppellF1} \left[\frac{3+m}{2}, -m, \right. \\
& \quad \left. 3+2m, \frac{5+m}{2}, \tan \left[\frac{1}{2} (a+b x) \right]^2, -\tan \left[\frac{1}{2} (a+b x) \right]^2 \right] \tan \left[\frac{1}{2} (a+b x) \right]^2 \right) \right. \\
& \quad \left. \frac{1}{1+m} 5 \times 2^{2+m} (3+m) \cos \left[\frac{1}{2} (a+b x) \right]^4 \sin \left[\frac{1}{2} (a+b x) \right]^2 \right)
\end{aligned}$$

$$\begin{aligned}
& \frac{5+m}{2}, \tan\left[\frac{1}{2}(a+b x)\right]^2, -\tan\left[\frac{1}{2}(a+b x)\right]^2] + 2(1+m) \operatorname{AppellF1}\left[\frac{3+m}{2}, -m, \right. \\
& \left. 3+2m, \frac{5+m}{2}, \tan\left[\frac{1}{2}(a+b x)\right]^2, -\tan\left[\frac{1}{2}(a+b x)\right]^2\right] \tan\left[\frac{1}{2}(a+b x)\right]^2\Big) + \\
& \frac{1}{1+m} 2^{3+m} (3+m) \cos\left[\frac{1}{2}(a+b x)\right]^5 \sin\left[\frac{1}{2}(a+b x)\right] \left(\cos\left[\frac{1}{2}(a+b x)\right]\right. \\
& \left.\left(-\sin\left[\frac{1}{2}(a+b x)\right] + \sin\left[\frac{3}{2}(a+b x)\right]\right)^m\right. \\
& \left.-\left(\left(-\frac{1}{3+m} m (1+m) \operatorname{AppellF1}\left[1+\frac{1+m}{2}, 1-m, 3+2m, 1+\frac{3+m}{2}, \tan\left[\frac{1}{2}(a+b x)\right]^2, \right. \right. \right. \right. \\
& \left. \left. \left. \left.-\tan\left[\frac{1}{2}(a+b x)\right]^2\right] \sec\left[\frac{1}{2}(a+b x)\right]^2 \tan\left[\frac{1}{2}(a+b x)\right] - \right. \right. \right. \\
& \left. \left. \left.\frac{1}{3+m} (1+m) (3+2m) \operatorname{AppellF1}\left[1+\frac{1+m}{2}, -m, 4+2m, 1+\frac{3+m}{2}, \right. \right. \right. \right. \\
& \left. \left. \left. \left.\tan\left[\frac{1}{2}(a+b x)\right]^2, -\tan\left[\frac{1}{2}(a+b x)\right]^2\right] \sec\left[\frac{1}{2}(a+b x)\right]^2 \tan\left[\frac{1}{2}(a+b x)\right]\right)\right) / \\
& \left((3+m) \operatorname{AppellF1}\left[\frac{1+m}{2}, -m, 3+2m, \frac{3+m}{2}, \tan\left[\frac{1}{2}(a+b x)\right]^2, -\tan\left[\frac{1}{2}(a+b x)\right]^2\right] - \right. \\
& \left. 2 \left(m \operatorname{AppellF1}\left[\frac{3+m}{2}, 1-m, 3+2m, \frac{5+m}{2}, \tan\left[\frac{1}{2}(a+b x)\right]^2, -\tan\left[\frac{1}{2}(a+b x)\right]^2\right] + \right. \right. \\
& \left. \left.(3+2m) \operatorname{AppellF1}\left[\frac{3+m}{2}, -m, 2(2+m), \frac{5+m}{2}, \right. \right. \right. \\
& \left. \left. \left.\tan\left[\frac{1}{2}(a+b x)\right]^2, -\tan\left[\frac{1}{2}(a+b x)\right]^2\right] \tan\left[\frac{1}{2}(a+b x)\right]^2\right) + \right. \\
& \left(\operatorname{AppellF1}\left[\frac{1+m}{2}, -m, 2(1+m), \frac{3+m}{2}, \tan\left[\frac{1}{2}(a+b x)\right]^2, -\tan\left[\frac{1}{2}(a+b x)\right]^2\right] \right. \\
& \left.\sec\left[\frac{1}{2}(a+b x)\right]^2 \tan\left[\frac{1}{2}(a+b x)\right]\right) / \\
& \left((3+m) \operatorname{AppellF1}\left[\frac{1+m}{2}, -m, 2(1+m), \frac{3+m}{2}, \tan\left[\frac{1}{2}(a+b x)\right]^2, -\tan\left[\frac{1}{2}(a+b x)\right]^2\right] - \right. \\
& \left. 2 \left(m \operatorname{AppellF1}\left[\frac{3+m}{2}, 1-m, 2(1+m), \frac{5+m}{2}, \tan\left[\frac{1}{2}(a+b x)\right]^2, \right. \right. \right. \\
& \left. \left. \left.-\tan\left[\frac{1}{2}(a+b x)\right]^2\right] + 2(1+m) \operatorname{AppellF1}\left[\frac{3+m}{2}, -m, 3+2m, \frac{5+m}{2}, \right. \right. \right. \\
& \left. \left. \left.\tan\left[\frac{1}{2}(a+b x)\right]^2, -\tan\left[\frac{1}{2}(a+b x)\right]^2\right] \tan\left[\frac{1}{2}(a+b x)\right]^2\right) + \right. \\
& \left(\sec\left[\frac{1}{2}(a+b x)\right]^2 \left(-\frac{1}{3+m} m (1+m) \operatorname{AppellF1}\left[1+\frac{1+m}{2}, 1-m, 2(1+m), 1+\frac{3+m}{2}, \right. \right. \right. \right. \\
& \left. \left. \left. \left.\tan\left[\frac{1}{2}(a+b x)\right]^2, -\tan\left[\frac{1}{2}(a+b x)\right]^2\right] \sec\left[\frac{1}{2}(a+b x)\right]^2 \tan\left[\frac{1}{2}(a+b x)\right] - \right. \right. \right. \\
& \left. \left. \left.\frac{1}{3+m} 2(1+m)^2 \operatorname{AppellF1}\left[1+\frac{1+m}{2}, -m, 1+2(1+m), 1+\frac{3+m}{2}, \right. \right. \right. \right. \\
& \left. \left. \left. \left.\tan\left[\frac{1}{2}(a+b x)\right]^2, -\tan\left[\frac{1}{2}(a+b x)\right]^2\right] \sec\left[\frac{1}{2}(a+b x)\right]^2 \tan\left[\frac{1}{2}(a+b x)\right]\right)\right) / \\
& \left((3+m) \operatorname{AppellF1}\left[\frac{1+m}{2}, -m, 2(1+m), \frac{3+m}{2}, \tan\left[\frac{1}{2}(a+b x)\right]^2, -\tan\left[\frac{1}{2}(a+b x)\right]^2\right] - \right.
\end{aligned}$$

Problem 125: Result unnecessarily involves complex numbers and more than

twice size of optimal antiderivative.

$$\int \sin[a + bx] \sin[2a + 2bx]^m dx$$

Optimal (type 5, 82 leaves, 2 steps):

$$\frac{1}{b(2+m)} (\cos[a+bx]^2)^{\frac{1-m}{2}} \text{Hypergeometric2F1}\left[\frac{1-m}{2}, \frac{2+m}{2}, \frac{4+m}{2}, \sin[a+bx]^2\right]$$

$$\sin[a+bx] \sin[2a+2bx]^m \tan[a+bx]$$

Result (type 5, 170 leaves):

$$\frac{1}{b(-1+4m^2)} 2^{-1-m} e^{-i(a+bx)} (1 - e^{4i(a+bx)})^{-m} (-i e^{-2i(a+bx)} (-1 + e^{4i(a+bx)}))^m$$

$$\left((1-2m) \text{Hypergeometric2F1}\left[\frac{1}{4} (-1-2m), -m, \frac{1}{4} (3-2m), e^{4i(a+bx)}\right] + \right.$$

$$\left. e^{2i(a+bx)} (1+2m) \text{Hypergeometric2F1}\left[\frac{1}{4} (1-2m), -m, \frac{1}{4} (5-2m), e^{4i(a+bx)}\right] \right)$$

Problem 126: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \csc[a+bx] \sin[2a+2bx]^m dx$$

Optimal (type 5, 72 leaves, 2 steps):

$$\frac{1}{b m} (\cos[a+bx]^2)^{\frac{1-m}{2}}$$

$$\text{Hypergeometric2F1}\left[\frac{1-m}{2}, \frac{m}{2}, \frac{2+m}{2}, \sin[a+bx]^2\right] \sec[a+bx] \sin[2a+2bx]^m$$

Result (type 6, 1737 leaves):

$$\left((2+m) \text{AppellF1}\left[\frac{m}{2}, -m, 2m, \frac{2+m}{2}, \tan\left[\frac{1}{2}(a+bx)\right]^2, -\tan\left[\frac{1}{2}(a+bx)\right]^2\right] \right.$$

$$\left. \csc[a+bx] \sin[2(a+bx)]^{2m} \right) /$$

$$\left(b m \left((2+m) \text{AppellF1}\left[\frac{m}{2}, -m, 2m, \frac{2+m}{2}, \tan\left[\frac{1}{2}(a+bx)\right]^2, -\tan\left[\frac{1}{2}(a+bx)\right]^2\right] - \right. \right.$$

$$2m \left(\text{AppellF1}\left[\frac{2+m}{2}, 1-m, 2m, \frac{4+m}{2}, \tan\left[\frac{1}{2}(a+bx)\right]^2, -\tan\left[\frac{1}{2}(a+bx)\right]^2\right] + 2 \text{AppellF1}\left[\frac{2+m}{2}, -m, 1+2m, \frac{4+m}{2}, \tan\left[\frac{1}{2}(a+bx)\right]^2, -\tan\left[\frac{1}{2}(a+bx)\right]^2\right] \right) \tan\left[\frac{1}{2}(a+bx)\right]^2 \right) -$$

$$\left(\left(2(2+m) \text{AppellF1}\left[\frac{m}{2}, -m, 2m, \frac{2+m}{2}, \tan\left[\frac{1}{2}(a+bx)\right]^2, -\tan\left[\frac{1}{2}(a+bx)\right]^2\right] \right. \right.$$

$$\left. \left. \cos[2(a+bx)] \sin[2(a+bx)]^{-1+m} \right) / \right.$$

$$\left((2+m) \text{AppellF1}\left[\frac{m}{2}, -m, 2m, \frac{2+m}{2}, \tan\left[\frac{1}{2}(a+bx)\right]^2, -\tan\left[\frac{1}{2}(a+bx)\right]^2\right] - \right)$$

$$\begin{aligned}
& 2m \left(\text{AppellF1} \left[\frac{2+m}{2}, 1-m, 2m, \frac{4+m}{2}, \tan \left[\frac{1}{2} (a+b x) \right]^2, -\tan \left[\frac{1}{2} (a+b x) \right]^2 \right] + \right. \\
& \quad \left. 2 \text{AppellF1} \left[\frac{2+m}{2}, -m, 1+2m, \frac{4+m}{2}, \tan \left[\frac{1}{2} (a+b x) \right]^2, -\tan \left[\frac{1}{2} (a+b x) \right]^2 \right] \right) \\
& \quad \tan \left[\frac{1}{2} (a+b x) \right]^2 + \left((2+m) \sin [2 (a+b x)]^m \right. \\
& \quad \left. \left(-\frac{1}{2+m} m^2 \text{AppellF1} \left[1+\frac{m}{2}, 1-m, 2m, 1+\frac{2+m}{2}, \tan \left[\frac{1}{2} (a+b x) \right]^2, -\tan \left[\frac{1}{2} (a+b x) \right]^2 \right] \right. \right. \\
& \quad \left. \left. \sec \left[\frac{1}{2} (a+b x) \right]^2 \tan \left[\frac{1}{2} (a+b x) \right] - \frac{1}{2+m} 2m^2 \text{AppellF1} \left[1+\frac{m}{2}, -m, 1+2m, 1+\frac{2+m}{2}, \right. \right. \\
& \quad \left. \left. \tan \left[\frac{1}{2} (a+b x) \right]^2, -\tan \left[\frac{1}{2} (a+b x) \right]^2 \right] \sec \left[\frac{1}{2} (a+b x) \right]^2 \tan \left[\frac{1}{2} (a+b x) \right] \right) \right) / \\
& \left(m \left((2+m) \text{AppellF1} \left[\frac{m}{2}, -m, 2m, \frac{2+m}{2}, \tan \left[\frac{1}{2} (a+b x) \right]^2, -\tan \left[\frac{1}{2} (a+b x) \right]^2 \right] - \right. \right. \\
& \quad \left. \left. 2m \left(\text{AppellF1} \left[\frac{2+m}{2}, 1-m, 2m, \frac{4+m}{2}, \tan \left[\frac{1}{2} (a+b x) \right]^2, -\tan \left[\frac{1}{2} (a+b x) \right]^2 \right] + \right. \right. \\
& \quad \left. \left. 2 \text{AppellF1} \left[\frac{2+m}{2}, -m, 1+2m, \frac{4+m}{2}, \tan \left[\frac{1}{2} (a+b x) \right]^2, -\tan \left[\frac{1}{2} (a+b x) \right]^2 \right] \right) \right. \\
& \quad \left. \tan \left[\frac{1}{2} (a+b x) \right]^2 \right) - \left((2+m) \text{AppellF1} \left[\frac{m}{2}, -m, 2m, \frac{2+m}{2}, \right. \right. \\
& \quad \left. \left. \tan \left[\frac{1}{2} (a+b x) \right]^2, -\tan \left[\frac{1}{2} (a+b x) \right]^2 \right] \sin [2 (a+b x)]^m \right. \\
& \quad \left. \left(-2m \left(\text{AppellF1} \left[\frac{2+m}{2}, 1-m, 2m, \frac{4+m}{2}, \tan \left[\frac{1}{2} (a+b x) \right]^2, -\tan \left[\frac{1}{2} (a+b x) \right]^2 \right] + \right. \right. \right. \\
& \quad \left. \left. \left. 2 \text{AppellF1} \left[\frac{2+m}{2}, -m, 1+2m, \frac{4+m}{2}, \tan \left[\frac{1}{2} (a+b x) \right]^2, -\tan \left[\frac{1}{2} (a+b x) \right]^2 \right] \right) \right. \\
& \quad \left. \sec \left[\frac{1}{2} (a+b x) \right]^2 \tan \left[\frac{1}{2} (a+b x) \right] + (2+m) \left(-\frac{1}{2+m} m^2 \text{AppellF1} \left[1+\frac{m}{2}, 1-m, \right. \right. \right. \\
& \quad \left. \left. \left. 2m, 1+\frac{2+m}{2}, \tan \left[\frac{1}{2} (a+b x) \right]^2, -\tan \left[\frac{1}{2} (a+b x) \right]^2 \right] \sec \left[\frac{1}{2} (a+b x) \right]^2 \right. \\
& \quad \left. \tan \left[\frac{1}{2} (a+b x) \right] - \frac{1}{2+m} 2m^2 \text{AppellF1} \left[1+\frac{m}{2}, -m, 1+2m, 1+\frac{2+m}{2}, \right. \right. \\
& \quad \left. \left. \tan \left[\frac{1}{2} (a+b x) \right]^2, -\tan \left[\frac{1}{2} (a+b x) \right]^2 \right] \sec \left[\frac{1}{2} (a+b x) \right]^2 \tan \left[\frac{1}{2} (a+b x) \right] \right) - \right. \\
& \quad \left. 2m \tan \left[\frac{1}{2} (a+b x) \right]^2 \left(-\frac{1}{4+m} 2m (2+m) \text{AppellF1} \left[1+\frac{2+m}{2}, 1-m, 1+2m, 1+\frac{4+m}{2}, \right. \right. \right. \\
& \quad \left. \left. \left. \tan \left[\frac{1}{2} (a+b x) \right]^2, -\tan \left[\frac{1}{2} (a+b x) \right]^2 \right] \sec \left[\frac{1}{2} (a+b x) \right]^2 \tan \left[\frac{1}{2} (a+b x) \right] + \right. \right. \\
& \quad \left. \left. \frac{1}{4+m} (1-m) (2+m) \text{AppellF1} \left[1+\frac{2+m}{2}, 2-m, 2m, 1+\frac{4+m}{2}, \tan \left[\frac{1}{2} (a+b x) \right]^2, \right. \right. \right. \\
& \quad \left. \left. \left. -\tan \left[\frac{1}{2} (a+b x) \right]^2 \right] \sec \left[\frac{1}{2} (a+b x) \right]^2 \tan \left[\frac{1}{2} (a+b x) \right] + \right. \right. \\
& \quad \left. \left. 2 \left(-\frac{1}{4+m} m (2+m) \text{AppellF1} \left[1+\frac{2+m}{2}, 1-m, 1+2m, 1+\frac{4+m}{2}, \tan \left[\frac{1}{2} (a+b x) \right]^2, \right. \right. \right. \\
& \quad \left. \left. \left. -\tan \left[\frac{1}{2} (a+b x) \right]^2 \right] \sec \left[\frac{1}{2} (a+b x) \right]^2 \tan \left[\frac{1}{2} (a+b x) \right] - \frac{1}{4+m} \right. \right. \right)
\end{aligned}$$

Problem 127: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \csc [a + b x]^2 \sin [2 a + 2 b x]^m dx$$

Optimal (type 5, 85 leaves, 2 steps):

$$-\frac{1}{b(1-m)} \left(\cos[a+bx]^2\right)^{\frac{1-m}{2}} \csc[a+bx] \text{Hypergeometric2F1}\left[\frac{1-m}{2}, \frac{1}{2}(-1+m), \frac{1+m}{2}, \sin[a+bx]^2\right] \sec[a+bx] \sin[2a+2bx]^m$$

Result (type 6, 4498 leaves):

$$\begin{aligned} & \left(2^{-1+m} \operatorname{Cot} \left[\frac{1}{2} (a + b x) \right] \operatorname{Csc}^2 [a + b x] \right. \\ & \left(\cos \left[\frac{1}{2} (a + b x) \right] \left(-\sin \left[\frac{1}{2} (a + b x) \right] + \sin \left[\frac{3}{2} (a + b x) \right] \right)^m \sin \left[2 (a + b x) \right]^m \right. \\ & \left(\left((1+m)^2 \operatorname{AppellF1} \left[\frac{1}{2} (-1+m), -m, 2m, \frac{1+m}{2}, \tan \left[\frac{1}{2} (a + b x) \right]^2, -\tan \left[\frac{1}{2} (a + b x) \right]^2 \right] \right) \right. \\ & \left((-1+m) \right. \\ & \left((1+m) \operatorname{AppellF1} \left[\frac{1}{2} (-1+m), -m, 2m, \frac{1+m}{2}, \tan \left[\frac{1}{2} (a + b x) \right]^2, -\tan \left[\frac{1}{2} (a + b x) \right]^2 \right] - \right. \\ & \left. 2m \left(\operatorname{AppellF1} \left[\frac{1+m}{2}, 1-m, 2m, \frac{3+m}{2}, \tan \left[\frac{1}{2} (a + b x) \right]^2, -\tan \left[\frac{1}{2} (a + b x) \right]^2 \right] + \right. \right. \\ & \left. \left. 2 \operatorname{AppellF1} \left[\frac{1+m}{2}, -m, 1+2m, \frac{3+m}{2}, \tan \left[\frac{1}{2} (a + b x) \right]^2, -\tan \left[\frac{1}{2} (a + b x) \right]^2 \right] \right) \right. \\ & \left. \tan \left[\frac{1}{2} (a + b x) \right]^2 \right) + \left((3+m) \operatorname{AppellF1} \left[\frac{1+m}{2}, -m, 2m, \frac{3+m}{2}, \right. \right. \\ & \left. \left. \tan \left[\frac{1}{2} (a + b x) \right]^2, -\tan \left[\frac{1}{2} (a + b x) \right]^2 \right] \tan \left[\frac{1}{2} (a + b x) \right]^2 \right) \right. \\ & \left((3+m) \operatorname{AppellF1} \left[\frac{1+m}{2}, -m, 2m, \frac{3+m}{2}, \tan \left[\frac{1}{2} (a + b x) \right]^2, -\tan \left[\frac{1}{2} (a + b x) \right]^2 \right] - \right. \end{aligned}$$

$$\begin{aligned}
& 2m \left(\text{AppellF1} \left[\frac{3+m}{2}, 1-m, 2m, \frac{5+m}{2}, \tan \left[\frac{1}{2} (a+b x) \right]^2, -\tan \left[\frac{1}{2} (a+b x) \right]^2 \right] + \right. \\
& \quad \left. 2 \text{AppellF1} \left[\frac{3+m}{2}, -m, 1+2m, \frac{5+m}{2}, \tan \left[\frac{1}{2} (a+b x) \right]^2, -\tan \left[\frac{1}{2} (a+b x) \right]^2 \right] \right) \\
& \quad \left(\tan \left[\frac{1}{2} (a+b x) \right]^2 \right) \Bigg) \Bigg) \Bigg/ \left(b (1+m) \right) \\
& \left(-\frac{1}{1+m} 2^{-2+m} \csc \left[\frac{1}{2} (a+b x) \right]^2 \left(\cos \left[\frac{1}{2} (a+b x) \right] \left(-\sin \left[\frac{1}{2} (a+b x) \right] + \sin \left[\frac{3}{2} (a+b x) \right] \right) \right)^m \right. \\
& \quad \left(\left((1+m)^2 \text{AppellF1} \left[\frac{1}{2} (-1+m), -m, 2m, \frac{1+m}{2}, \tan \left[\frac{1}{2} (a+b x) \right]^2, \right. \right. \right. \\
& \quad \left. \left. \left. -\tan \left[\frac{1}{2} (a+b x) \right]^2 \right] \right) \Bigg/ \left((-1+m) \left((1+m) \text{AppellF1} \left[\frac{1}{2} (-1+m), -m, 2m, \frac{1+m}{2}, \right. \right. \right. \right. \\
& \quad \left. \left. \left. \tan \left[\frac{1}{2} (a+b x) \right]^2, -\tan \left[\frac{1}{2} (a+b x) \right]^2 \right] - 2m \left(\text{AppellF1} \left[\frac{1+m}{2}, 1-m, 2m, \frac{3+m}{2}, \right. \right. \right. \\
& \quad \left. \left. \left. \tan \left[\frac{1}{2} (a+b x) \right]^2, -\tan \left[\frac{1}{2} (a+b x) \right]^2 \right] + 2 \text{AppellF1} \left[\frac{1+m}{2}, -m, 1+2m, \right. \right. \right. \\
& \quad \left. \left. \left. \frac{3+m}{2}, \tan \left[\frac{1}{2} (a+b x) \right]^2, -\tan \left[\frac{1}{2} (a+b x) \right]^2 \right] \right) \tan \left[\frac{1}{2} (a+b x) \right]^2 \right) + \\
& \quad \left((3+m) \text{AppellF1} \left[\frac{1+m}{2}, -m, 2m, \frac{3+m}{2}, \tan \left[\frac{1}{2} (a+b x) \right]^2, -\tan \left[\frac{1}{2} (a+b x) \right]^2 \right] \right. \\
& \quad \left. \tan \left[\frac{1}{2} (a+b x) \right]^2 \right) \Bigg/ \left((3+m) \text{AppellF1} \left[\frac{1+m}{2}, -m, 2m, \frac{3+m}{2}, \tan \left[\frac{1}{2} (a+b x) \right]^2, \right. \right. \\
& \quad \left. \left. -\tan \left[\frac{1}{2} (a+b x) \right]^2 \right] - 2m \left(\text{AppellF1} \left[\frac{3+m}{2}, 1-m, 2m, \frac{5+m}{2}, \right. \right. \right. \\
& \quad \left. \left. \left. \tan \left[\frac{1}{2} (a+b x) \right]^2, -\tan \left[\frac{1}{2} (a+b x) \right]^2 \right] + 2 \text{AppellF1} \left[\frac{3+m}{2}, -m, 1+2m, \right. \right. \right. \\
& \quad \left. \left. \left. \frac{5+m}{2}, \tan \left[\frac{1}{2} (a+b x) \right]^2, -\tan \left[\frac{1}{2} (a+b x) \right]^2 \right] \right) \tan \left[\frac{1}{2} (a+b x) \right]^2 \right) + \\
& \quad \left. \frac{1}{1+m} 2^{-1+m} m \cot \left[\frac{1}{2} (a+b x) \right] \left(\cos \left[\frac{1}{2} (a+b x) \right] \left(-\sin \left[\frac{1}{2} (a+b x) \right] + \sin \left[\frac{3}{2} (a+b x) \right] \right) \right)^{-1+m} \right. \\
& \quad \left(\cos \left[\frac{1}{2} (a+b x) \right] \left(-\frac{1}{2} \cos \left[\frac{1}{2} (a+b x) \right] + \frac{3}{2} \cos \left[\frac{3}{2} (a+b x) \right] \right) - \right. \\
& \quad \left. \frac{1}{2} \sin \left[\frac{1}{2} (a+b x) \right] \left(-\sin \left[\frac{1}{2} (a+b x) \right] + \sin \left[\frac{3}{2} (a+b x) \right] \right) \right) \\
& \quad \left(\left((1+m)^2 \text{AppellF1} \left[\frac{1}{2} (-1+m), -m, 2m, \frac{1+m}{2}, \tan \left[\frac{1}{2} (a+b x) \right]^2, -\tan \left[\frac{1}{2} (a+b x) \right]^2 \right] \right) \right. \\
& \quad \left((-1+m) \left((1+m) \text{AppellF1} \left[\frac{1}{2} (-1+m), -m, 2m, \frac{1+m}{2}, \tan \left[\frac{1}{2} (a+b x) \right]^2, \right. \right. \right. \\
& \quad \left. \left. \left. -\tan \left[\frac{1}{2} (a+b x) \right]^2 \right] - 2m \left(\text{AppellF1} \left[\frac{1+m}{2}, 1-m, 2m, \frac{3+m}{2}, \right. \right. \right. \\
& \quad \left. \left. \left. \tan \left[\frac{1}{2} (a+b x) \right]^2, -\tan \left[\frac{1}{2} (a+b x) \right]^2 \right] + 2 \text{AppellF1} \left[\frac{1+m}{2}, -m, 1+2m, \right. \right. \right. \\
& \quad \left. \left. \left. \frac{3+m}{2}, \tan \left[\frac{1}{2} (a+b x) \right]^2, -\tan \left[\frac{1}{2} (a+b x) \right]^2 \right] \right) \tan \left[\frac{1}{2} (a+b x) \right]^2 \right) + \\
& \quad \left((3+m) \text{AppellF1} \left[\frac{1+m}{2}, -m, 2m, \frac{3+m}{2}, \tan \left[\frac{1}{2} (a+b x) \right]^2, -\tan \left[\frac{1}{2} (a+b x) \right]^2 \right] \right)
\end{aligned}$$

$$\begin{aligned}
& \left. \frac{\tan\left(\frac{1}{2}(a+b x)\right)^2}{\left((3+m) \operatorname{AppellF1}\left[\frac{1+m}{2}, -m, 2 m, \frac{3+m}{2}, \tan\left(\frac{1}{2}(a+b x)\right)^2,\right.\right.} \right. \\
& \left. \left.-\tan\left(\frac{1}{2}(a+b x)\right)^2\right]-2 m \left(\operatorname{AppellF1}\left[\frac{3+m}{2}, 1-m, 2 m, \frac{5+m}{2},\right.\right. \\
& \left. \left.\tan\left(\frac{1}{2}(a+b x)\right)^2, -\tan\left(\frac{1}{2}(a+b x)\right)^2\right]+2 \operatorname{AppellF1}\left[\frac{3+m}{2}, -m, 1+2 m,\right.\right. \\
& \left. \left.\frac{5+m}{2}, \tan\left(\frac{1}{2}(a+b x)\right)^2, -\tan\left(\frac{1}{2}(a+b x)\right)^2\right)\right] \tan\left(\frac{1}{2}(a+b x)\right)^2\right)+ \\
& \frac{1}{1+m} 2^{-1+m} \cot\left(\frac{1}{2}(a+b x)\right)\left(\cos\left(\frac{1}{2}(a+b x)\right)\left(-\sin\left(\frac{1}{2}(a+b x)\right)+\sin\left(\frac{3}{2}(a+b x)\right)\right)\right)^m \\
& \left(\left((1+m)^2\left(-\frac{1}{1+m}(-1+m) m \operatorname{AppellF1}\left[1+\frac{1}{2}(-1+m), 1-m, 2 m, 1+\frac{1+m}{2},\right.\right.\right.\right. \\
& \left.\left.\left.\left.\tan\left(\frac{1}{2}(a+b x)\right)^2, -\tan\left(\frac{1}{2}(a+b x)\right)^2\right] \sec\left(\frac{1}{2}(a+b x)\right)^2 \tan\left(\frac{1}{2}(a+b x)\right]\right)-\right. \\
& \left.\left.\frac{1}{1+m} 2(-1+m) m \operatorname{AppellF1}\left[1+\frac{1}{2}(-1+m), -m, 1+2 m, 1+\frac{1+m}{2},\right.\right.\right. \\
& \left.\left.\left.\tan\left(\frac{1}{2}(a+b x)\right)^2, -\tan\left(\frac{1}{2}(a+b x)\right)^2\right] \sec\left(\frac{1}{2}(a+b x)\right)^2 \tan\left(\frac{1}{2}(a+b x)\right]\right)\right) \\
& \left.\left((-1+m)\left((1+m) \operatorname{AppellF1}\left[\frac{1}{2}(-1+m), -m, 2 m, \frac{1+m}{2}, \tan\left(\frac{1}{2}(a+b x)\right)^2,\right.\right.\right.\right. \\
& \left.\left.\left.\left.-\tan\left(\frac{1}{2}(a+b x)\right)^2\right]-2 m \left(\operatorname{AppellF1}\left[\frac{1+m}{2}, 1-m, 2 m, \frac{3+m}{2},\right.\right.\right.\right. \\
& \left.\left.\left.\left.\tan\left(\frac{1}{2}(a+b x)\right)^2, -\tan\left(\frac{1}{2}(a+b x)\right)^2\right]+2 \operatorname{AppellF1}\left[\frac{1+m}{2}, -m, 1+2 m,\right.\right.\right.\right. \\
& \left.\left.\left.\left.\frac{3+m}{2}, \tan\left(\frac{1}{2}(a+b x)\right)^2, -\tan\left(\frac{1}{2}(a+b x)\right)^2\right)\right] \tan\left(\frac{1}{2}(a+b x)\right)^2\right)+ \\
& \left.\left((3+m) \operatorname{AppellF1}\left[\frac{1+m}{2}, -m, 2 m, \frac{3+m}{2}, \tan\left(\frac{1}{2}(a+b x)\right)^2, -\tan\left(\frac{1}{2}(a+b x)\right)^2\right]\right. \\
& \left.\left.\sec\left(\frac{1}{2}(a+b x)\right)^2 \tan\left(\frac{1}{2}(a+b x)\right)\right)\right) \\
& \left.\left((3+m) \operatorname{AppellF1}\left[\frac{1+m}{2}, -m, 2 m, \frac{3+m}{2}, \tan\left(\frac{1}{2}(a+b x)\right)^2, -\tan\left(\frac{1}{2}(a+b x)\right)^2\right]-\right. \right. \\
& \left.\left.2 m \left(\operatorname{AppellF1}\left[\frac{3+m}{2}, 1-m, 2 m, \frac{5+m}{2}, \tan\left(\frac{1}{2}(a+b x)\right)^2,\right.\right.\right.\right. \\
& \left.\left.\left.\left.-\tan\left(\frac{1}{2}(a+b x)\right)^2\right]+2 \operatorname{AppellF1}\left[\frac{3+m}{2}, -m, 1+2 m, \frac{5+m}{2},\right.\right.\right.\right. \\
& \left.\left.\left.\left.\tan\left(\frac{1}{2}(a+b x)\right)^2, -\tan\left(\frac{1}{2}(a+b x)\right)^2\right)\right] \tan\left(\frac{1}{2}(a+b x)\right)^2\right)+ \\
& \left.\left((3+m) \tan\left(\frac{1}{2}(a+b x)\right)^2\left(-\frac{1}{3+m} m(1+m) \operatorname{AppellF1}\left[1+\frac{1+m}{2}, 1-m, 2 m, 1+\frac{3+m}{2},\right.\right.\right.\right. \\
& \left.\left.\left.\left.\tan\left(\frac{1}{2}(a+b x)\right)^2, -\tan\left(\frac{1}{2}(a+b x)\right)^2\right] \sec\left(\frac{1}{2}(a+b x)\right)^2 \tan\left(\frac{1}{2}(a+b x)\right]\right)-\right. \\
& \left.\left.\frac{1}{3+m} 2 m(1+m) \operatorname{AppellF1}\left[1+\frac{1+m}{2}, -m, 1+2 m, 1+\frac{3+m}{2}, \tan\left(\frac{1}{2}(a+b x)\right)^2,\right.\right.\right.\right. \\
& \left.\left.\left.\left.-\tan\left(\frac{1}{2}(a+b x)\right)^2\right] \sec\left(\frac{1}{2}(a+b x)\right)^2 \tan\left(\frac{1}{2}(a+b x)\right]\right)\right)
\end{aligned}$$

Problem 128: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \csc[a + bx]^3 \sin[2a + 2bx]^m dx$$

Optimal (type 5, 85 leaves, 2 steps):

$$-\frac{1}{b(2-m)} (\cos[a+bx]^2)^{\frac{1-m}{2}} \csc[a+bx]^2 \\ \text{Hypergeometric2F1}\left[\frac{1-m}{2}, \frac{1}{2}(-2+m), \frac{m}{2}, \sin[a+bx]^2\right] \sec[a+bx] \sin[2a+2bx]^m$$

Result (type 6, 5872 leaves):

$$\begin{aligned} & \left(4^{-1+m} \csc[a+bx]^3 \sin[2(a+bx)]^m \right. \\ & \left(\frac{\tan[\frac{1}{2}(a+bx)] - \tan[\frac{1}{2}(a+bx)]^3}{(1+\tan[\frac{1}{2}(a+bx)])^2} \right)^m \left(- \left(\left(\text{AppellF1}\left[\frac{1}{2}(-2+m), -m, \right. \right. \right. \right. \\ & \left. \left. \left. \left. 2m, \frac{m}{2}, \tan[\frac{1}{2}(a+bx)]^2, -\tan[\frac{1}{2}(a+bx)]^2 \right] \cot[\frac{1}{2}(a+bx)]^2 \right) \right. \\ & \left((-2+m) \left(-\text{AppellF1}\left[\frac{1}{2}(-2+m), -m, 2m, \frac{m}{2}, \tan[\frac{1}{2}(a+bx)]^2, -\tan[\frac{1}{2}(a+bx)]^2 \right] + \right. \right. \\ & \left. \left. 2 \left(\text{AppellF1}\left[\frac{m}{2}, 1-m, 2m, \frac{2+m}{2}, \tan[\frac{1}{2}(a+bx)]^2, -\tan[\frac{1}{2}(a+bx)]^2 \right] + 2 \right. \right. \\ & \left. \left. \text{AppellF1}\left[\frac{m}{2}, -m, 1+2m, \frac{2+m}{2}, \tan[\frac{1}{2}(a+bx)]^2, -\tan[\frac{1}{2}(a+bx)]^2 \right] \right) \right. \\ & \left. \left. \left. \left. \left(-\tan[\frac{1}{2}(a+bx)]^2 \right) \tan[\frac{1}{2}(a+bx)]^2 \right) \right) \right. \\ & \left. \left(2(2+m) \text{AppellF1}\left[\frac{m}{2}, -m, 2m, \frac{2+m}{2}, \tan[\frac{1}{2}(a+bx)]^2, -\tan[\frac{1}{2}(a+bx)]^2 \right] \right) \right. \\ & \left. \left(m \left((2+m) \text{AppellF1}\left[\frac{m}{2}, -m, 2m, \frac{2+m}{2}, \tan[\frac{1}{2}(a+bx)]^2, -\tan[\frac{1}{2}(a+bx)]^2 \right] - \right. \right. \right. \\ & \left. \left. \left. 2m \left(\text{AppellF1}\left[\frac{2+m}{2}, 1-m, 2m, \frac{4+m}{2}, \tan[\frac{1}{2}(a+bx)]^2, -\tan[\frac{1}{2}(a+bx)]^2 \right] + \right. \right. \right. \\ & \left. \left. \left. 2 \text{AppellF1}\left[\frac{2+m}{2}, -m, 1+2m, \frac{4+m}{2}, \tan[\frac{1}{2}(a+bx)]^2, -\tan[\frac{1}{2}(a+bx)]^2 \right] \right. \right. \right. \\ & \left. \left. \left. \left. \left(-\tan[\frac{1}{2}(a+bx)]^2 \right) \tan[\frac{1}{2}(a+bx)]^2 \right) \right) \right. \\ & \left. \left((4+m) \text{AppellF1}\left[\frac{2+m}{2}, -m, 2m, \frac{4+m}{2}, \tan[\frac{1}{2}(a+bx)]^2, -\tan[\frac{1}{2}(a+bx)]^2 \right] \right. \right. \\ & \left. \left. \left. \left. \left(\tan[\frac{1}{2}(a+bx)]^2 \right) \right) \right) \right. \\ & \left. \left((2+m) \left((4+m) \text{AppellF1}\left[\frac{2+m}{2}, -m, 2m, \frac{4+m}{2}, \tan[\frac{1}{2}(a+bx)]^2, -\tan[\frac{1}{2}(a+bx)]^2 \right] - \right. \right. \right. \\ & \left. \left. \left. 2m \left(\text{AppellF1}\left[\frac{4+m}{2}, 1-m, 2m, \frac{6+m}{2}, \tan[\frac{1}{2}(a+bx)]^2, -\tan[\frac{1}{2}(a+bx)]^2 \right] + \right. \right. \right. \\ & \left. \left. \left. 2 \text{AppellF1}\left[\frac{4+m}{2}, -m, 1+2m, \frac{6+m}{2}, \tan[\frac{1}{2}(a+bx)]^2, -\tan[\frac{1}{2}(a+bx)]^2 \right] \right) \right) \right) \end{aligned}$$

$$\begin{aligned}
& \left(- \left(\left(\text{AppellF1} \left[\frac{1}{2} (-2 + m), -m, 2m, \frac{m}{2}, \tan \left[\frac{1}{2} (a + b x) \right]^2, -\tan \left[\frac{1}{2} (a + b x) \right]^2 \right] \right) \right) \right) / \left(b \left(4^{-1+m} m \left(\frac{\tan \left[\frac{1}{2} (a + b x) \right] - \tan \left[\frac{1}{2} (a + b x) \right]^3}{\left(1 + \tan \left[\frac{1}{2} (a + b x) \right]^2 \right)^2} \right)^{-1+m} \right. \right. \\
& \left. \left. \left(- \left(\left(\text{AppellF1} \left[\frac{1}{2} (-2 + m), -m, 2m, \frac{m}{2}, \tan \left[\frac{1}{2} (a + b x) \right]^2, -\tan \left[\frac{1}{2} (a + b x) \right]^2 \right] \right) \right) \right) \right) / \left(\left(-2 + m \right) \left(-\text{AppellF1} \left[\frac{1}{2} (-2 + m), -m, 2m, \frac{m}{2}, \tan \left[\frac{1}{2} (a + b x) \right]^2, -\tan \left[\frac{1}{2} (a + b x) \right]^2 \right] + 2 \text{AppellF1} \left[\frac{m}{2}, 1-m, 2m, \frac{2+m}{2}, \tan \left[\frac{1}{2} (a + b x) \right]^2, -\tan \left[\frac{1}{2} (a + b x) \right]^2 \right] + 2 \text{AppellF1} \left[\frac{m}{2}, -m, 1+2m, \frac{2+m}{2}, \tan \left[\frac{1}{2} (a + b x) \right]^2, -\tan \left[\frac{1}{2} (a + b x) \right]^2 \right] \right) \tan \left[\frac{1}{2} (a + b x) \right]^2 \right) \right) + \\
& \left(2 (2+m) \text{AppellF1} \left[\frac{m}{2}, -m, 2m, \frac{2+m}{2}, \tan \left[\frac{1}{2} (a + b x) \right]^2, -\tan \left[\frac{1}{2} (a + b x) \right]^2 \right] \right) / \\
& \left(m \left((2+m) \text{AppellF1} \left[\frac{m}{2}, -m, 2m, \frac{2+m}{2}, \tan \left[\frac{1}{2} (a + b x) \right]^2, -\tan \left[\frac{1}{2} (a + b x) \right]^2 \right] - 2m \left(\text{AppellF1} \left[\frac{2+m}{2}, 1-m, 2m, \frac{4+m}{2}, \tan \left[\frac{1}{2} (a + b x) \right]^2, -\tan \left[\frac{1}{2} (a + b x) \right]^2 \right] + 2 \text{AppellF1} \left[\frac{2+m}{2}, -m, 1+2m, \frac{4+m}{2}, \tan \left[\frac{1}{2} (a + b x) \right]^2, -\tan \left[\frac{1}{2} (a + b x) \right]^2 \right) \tan \left[\frac{1}{2} (a + b x) \right]^2 \right) \right) + \\
& \left((4+m) \text{AppellF1} \left[\frac{2+m}{2}, -m, 2m, \frac{4+m}{2}, \tan \left[\frac{1}{2} (a + b x) \right]^2, -\tan \left[\frac{1}{2} (a + b x) \right]^2 \right] \tan \left[\frac{1}{2} (a + b x) \right]^2 \right) / \left((2+m) \left((4+m) \text{AppellF1} \left[\frac{2+m}{2}, -m, 2m, \frac{4+m}{2}, \tan \left[\frac{1}{2} (a + b x) \right]^2, -\tan \left[\frac{1}{2} (a + b x) \right]^2 \right] - 2m \left(\text{AppellF1} \left[\frac{4+m}{2}, 1-m, 2m, \frac{6+m}{2}, \tan \left[\frac{1}{2} (a + b x) \right]^2, -\tan \left[\frac{1}{2} (a + b x) \right]^2 \right] + 2 \text{AppellF1} \left[\frac{4+m}{2}, -m, 1+2m, \frac{6+m}{2}, \tan \left[\frac{1}{2} (a + b x) \right]^2, -\tan \left[\frac{1}{2} (a + b x) \right]^2 \right) \tan \left[\frac{1}{2} (a + b x) \right]^2 \right) \right) \right) \\
& \left(\frac{1}{2} \sec \left[\frac{1}{2} (a + b x) \right]^2 - \frac{3}{2} \sec \left[\frac{1}{2} (a + b x) \right]^2 \tan \left[\frac{1}{2} (a + b x) \right]^2 \right) / \left(1 + \tan \left[\frac{1}{2} (a + b x) \right]^2 \right)^2 - \\
& \left(2 \sec \left[\frac{1}{2} (a + b x) \right]^2 \tan \left[\frac{1}{2} (a + b x) \right] \left(\tan \left[\frac{1}{2} (a + b x) \right] - \tan \left[\frac{1}{2} (a + b x) \right]^3 \right) \right) / \\
& \left(1 + \tan \left[\frac{1}{2} (a + b x) \right]^2 \right)^3 + \\
& 4^{-1+m} \left(\frac{\tan \left[\frac{1}{2} (a + b x) \right] - \tan \left[\frac{1}{2} (a + b x) \right]^3}{\left(1 + \tan \left[\frac{1}{2} (a + b x) \right]^2 \right)^2} \right)^m
\end{aligned}$$

$$\begin{aligned}
& \left(\left(\text{AppellF1} \left[\frac{1}{2} (-2+m), -m, 2m, \frac{m}{2}, \tan \left[\frac{1}{2} (a+b x) \right]^2, -\tan \left[\frac{1}{2} (a+b x) \right]^2 \right] \right. \right. \\
& \quad \left. \left. \cot \left[\frac{1}{2} (a+b x) \right] \csc \left[\frac{1}{2} (a+b x) \right]^2 \right) / \right. \\
& \quad \left((-2+m) \left(-\text{AppellF1} \left[\frac{1}{2} (-2+m), -m, 2m, \frac{m}{2}, \tan \left[\frac{1}{2} (a+b x) \right]^2, -\tan \left[\frac{1}{2} (a+b x) \right]^2 \right] + \right. \right. \\
& \quad \left. \left. 2 \left(\text{AppellF1} \left[\frac{m}{2}, 1-m, 2m, \frac{2+m}{2}, \tan \left[\frac{1}{2} (a+b x) \right]^2, -\tan \left[\frac{1}{2} (a+b x) \right]^2 \right] + \right. \right. \\
& \quad \left. \left. 2 \text{AppellF1} \left[\frac{m}{2}, -m, 1+2m, \frac{2+m}{2}, \tan \left[\frac{1}{2} (a+b x) \right]^2, \right. \right. \\
& \quad \left. \left. -\tan \left[\frac{1}{2} (a+b x) \right]^2 \right] \right) \tan \left[\frac{1}{2} (a+b x) \right]^2 \right) - \\
& \quad \left(\cot \left[\frac{1}{2} (a+b x) \right]^2 \left(-(-2+m) \text{AppellF1} \left[1+\frac{1}{2} (-2+m), 1-m, 2m, 1+\frac{m}{2}, \right. \right. \right. \\
& \quad \left. \left. \tan \left[\frac{1}{2} (a+b x) \right]^2, -\tan \left[\frac{1}{2} (a+b x) \right]^2 \right] \sec \left[\frac{1}{2} (a+b x) \right]^2 \tan \left[\frac{1}{2} (a+b x) \right] - \right. \\
& \quad \left. \left. 2 (-2+m) \text{AppellF1} \left[1+\frac{1}{2} (-2+m), -m, 1+2m, 1+\frac{m}{2}, \tan \left[\frac{1}{2} (a+b x) \right]^2, \right. \right. \\
& \quad \left. \left. -\tan \left[\frac{1}{2} (a+b x) \right]^2 \right] \sec \left[\frac{1}{2} (a+b x) \right]^2 \tan \left[\frac{1}{2} (a+b x) \right] \right) \right) / \\
& \quad \left((-2+m) \left(-\text{AppellF1} \left[\frac{1}{2} (-2+m), -m, 2m, \frac{m}{2}, \tan \left[\frac{1}{2} (a+b x) \right]^2, -\tan \left[\frac{1}{2} (a+b x) \right]^2 \right] + \right. \right. \\
& \quad \left. \left. 2 \left(\text{AppellF1} \left[\frac{m}{2}, 1-m, 2m, \frac{2+m}{2}, \tan \left[\frac{1}{2} (a+b x) \right]^2, -\tan \left[\frac{1}{2} (a+b x) \right]^2 \right] + \right. \right. \\
& \quad \left. \left. 2 \text{AppellF1} \left[\frac{m}{2}, -m, 1+2m, \frac{2+m}{2}, \tan \left[\frac{1}{2} (a+b x) \right]^2, \right. \right. \\
& \quad \left. \left. -\tan \left[\frac{1}{2} (a+b x) \right]^2 \right] \right) \tan \left[\frac{1}{2} (a+b x) \right]^2 \right) + \\
& \quad \left(2 (2+m) \left(-\frac{1}{2+m} m^2 \text{AppellF1} \left[1+\frac{m}{2}, 1-m, 2m, 1+\frac{2+m}{2}, \tan \left[\frac{1}{2} (a+b x) \right]^2, \right. \right. \right. \\
& \quad \left. \left. -\tan \left[\frac{1}{2} (a+b x) \right]^2 \right] \sec \left[\frac{1}{2} (a+b x) \right]^2 \tan \left[\frac{1}{2} (a+b x) \right] - \right. \\
& \quad \left. \left. \frac{1}{2+m} 2m^2 \text{AppellF1} \left[1+\frac{m}{2}, -m, 1+2m, 1+\frac{2+m}{2}, \tan \left[\frac{1}{2} (a+b x) \right]^2, \right. \right. \\
& \quad \left. \left. -\tan \left[\frac{1}{2} (a+b x) \right]^2 \right] \sec \left[\frac{1}{2} (a+b x) \right]^2 \tan \left[\frac{1}{2} (a+b x) \right] \right) \right) / \\
& \quad \left(m \left((2+m) \text{AppellF1} \left[\frac{m}{2}, -m, 2m, \frac{2+m}{2}, \tan \left[\frac{1}{2} (a+b x) \right]^2, -\tan \left[\frac{1}{2} (a+b x) \right]^2 \right] - \right. \right. \\
& \quad \left. \left. 2m \left(\text{AppellF1} \left[\frac{2+m}{2}, 1-m, 2m, \frac{4+m}{2}, \tan \left[\frac{1}{2} (a+b x) \right]^2, -\tan \left[\frac{1}{2} (a+b x) \right]^2 \right] + \right. \right. \\
& \quad \left. \left. 2 \text{AppellF1} \left[\frac{2+m}{2}, -m, 1+2m, \frac{4+m}{2}, \tan \left[\frac{1}{2} (a+b x) \right]^2, \right. \right. \\
& \quad \left. \left. -\tan \left[\frac{1}{2} (a+b x) \right]^2 \right] \right) \tan \left[\frac{1}{2} (a+b x) \right]^2 \right) + \\
& \quad \left((4+m) \text{AppellF1} \left[\frac{2+m}{2}, -m, 2m, \frac{4+m}{2}, \tan \left[\frac{1}{2} (a+b x) \right]^2, -\tan \left[\frac{1}{2} (a+b x) \right]^2 \right] \right)
\end{aligned}$$

$$\begin{aligned}
& \left. \left(\sec \left[\frac{1}{2} (a + b x) \right]^2 \tan \left[\frac{1}{2} (a + b x) \right] \right) \right/ \left((2 + m) \right. \\
& \left. \left((4 + m) \text{AppellF1} \left[\frac{2+m}{2}, -m, 2m, \frac{4+m}{2}, \tan \left[\frac{1}{2} (a + b x) \right]^2, -\tan \left[\frac{1}{2} (a + b x) \right]^2 \right] - \right. \right. \\
& 2m \left(\text{AppellF1} \left[\frac{4+m}{2}, 1-m, 2m, \frac{6+m}{2}, \tan \left[\frac{1}{2} (a + b x) \right]^2, -\tan \left[\frac{1}{2} (a + b x) \right]^2 \right] + \right. \\
& 2 \text{AppellF1} \left[\frac{4+m}{2}, -m, 1+2m, \frac{6+m}{2}, \tan \left[\frac{1}{2} (a + b x) \right]^2, \right. \\
& \left. \left. \left. -\tan \left[\frac{1}{2} (a + b x) \right]^2 \right] \tan \left[\frac{1}{2} (a + b x) \right]^2 \right) \right) + \\
& \left. \left((4 + m) \tan \left[\frac{1}{2} (a + b x) \right]^2 \left(-\frac{1}{4+m}m (2 + m) \text{AppellF1} \left[1 + \frac{2+m}{2}, 1-m, 2m, 1 + \frac{4+m}{2}, \right. \right. \right. \right. \\
& \tan \left[\frac{1}{2} (a + b x) \right]^2, -\tan \left[\frac{1}{2} (a + b x) \right]^2 \right) \sec \left[\frac{1}{2} (a + b x) \right]^2 \tan \left[\frac{1}{2} (a + b x) \right] - \\
& \frac{1}{4+m} 2m (2 + m) \text{AppellF1} \left[1 + \frac{2+m}{2}, -m, 1+2m, 1 + \frac{4+m}{2}, \tan \left[\frac{1}{2} (a + b x) \right]^2, \right. \\
& \left. \left. \left. -\tan \left[\frac{1}{2} (a + b x) \right]^2 \right] \sec \left[\frac{1}{2} (a + b x) \right]^2 \tan \left[\frac{1}{2} (a + b x) \right] \right) \right) \right/ \\
& \left((2 + m) \left((4 + m) \text{AppellF1} \left[\frac{2+m}{2}, -m, 2m, \frac{4+m}{2}, \tan \left[\frac{1}{2} (a + b x) \right]^2, \right. \right. \right. \right. \\
& -\tan \left[\frac{1}{2} (a + b x) \right]^2 \right] - 2m \left(\text{AppellF1} \left[\frac{4+m}{2}, 1-m, 2m, \frac{6+m}{2}, \right. \right. \right. \\
& \tan \left[\frac{1}{2} (a + b x) \right]^2, -\tan \left[\frac{1}{2} (a + b x) \right]^2 \right] + 2 \text{AppellF1} \left[\frac{4+m}{2}, -m, 1+2m, \right. \\
& \left. \left. \left. \frac{6+m}{2}, \tan \left[\frac{1}{2} (a + b x) \right]^2, -\tan \left[\frac{1}{2} (a + b x) \right]^2 \right] \tan \left[\frac{1}{2} (a + b x) \right]^2 \right) \right) + \\
& \left(\text{AppellF1} \left[\frac{1}{2} (-2+m), -m, 2m, \frac{m}{2}, \tan \left[\frac{1}{2} (a + b x) \right]^2, -\tan \left[\frac{1}{2} (a + b x) \right]^2 \right] \right. \\
& \cot \left[\frac{1}{2} (a + b x) \right]^2 \left((-2 + m) \text{AppellF1} \left[1 + \frac{1}{2} (-2+m), 1-m, 2m, 1 + \frac{m}{2}, \right. \right. \\
& \tan \left[\frac{1}{2} (a + b x) \right]^2, -\tan \left[\frac{1}{2} (a + b x) \right]^2 \right) \sec \left[\frac{1}{2} (a + b x) \right]^2 \tan \left[\frac{1}{2} (a + b x) \right] + \\
& 2 (-2 + m) \text{AppellF1} \left[1 + \frac{1}{2} (-2+m), -m, 1+2m, 1 + \frac{m}{2}, \tan \left[\frac{1}{2} (a + b x) \right]^2, \right. \\
& \left. -\tan \left[\frac{1}{2} (a + b x) \right]^2 \right] \sec \left[\frac{1}{2} (a + b x) \right]^2 \tan \left[\frac{1}{2} (a + b x) \right] + \\
& 2 \left(\text{AppellF1} \left[\frac{m}{2}, 1-m, 2m, \frac{2+m}{2}, \tan \left[\frac{1}{2} (a + b x) \right]^2, -\tan \left[\frac{1}{2} (a + b x) \right]^2 \right] + \right. \\
& \left. 2 \text{AppellF1} \left[\frac{m}{2}, -m, 1+2m, \frac{2+m}{2}, \tan \left[\frac{1}{2} (a + b x) \right]^2, -\tan \left[\frac{1}{2} (a + b x) \right]^2 \right] \right) \\
& \sec \left[\frac{1}{2} (a + b x) \right]^2 \tan \left[\frac{1}{2} (a + b x) \right] + 2 \tan \left[\frac{1}{2} (a + b x) \right]^2 \\
& \left(-\frac{1}{2+m} 2m^2 \text{AppellF1} \left[1 + \frac{m}{2}, 1-m, 1+2m, 1 + \frac{2+m}{2}, \tan \left[\frac{1}{2} (a + b x) \right]^2, \right. \right. \\
& \left. -\tan \left[\frac{1}{2} (a + b x) \right]^2 \right) \sec \left[\frac{1}{2} (a + b x) \right]^2 \tan \left[\frac{1}{2} (a + b x) \right] + \frac{1}{2+m}
\end{aligned}$$

$$\begin{aligned}
& \left(1 - m\right) m \operatorname{AppellF1}\left[1 + \frac{m}{2}, 2 - m, 2m, 1 + \frac{2+m}{2}, \tan\left[\frac{1}{2}(a+b x)\right]^2, \right. \\
& \quad \left.- \tan\left[\frac{1}{2}(a+b x)\right]^2\right] \sec\left[\frac{1}{2}(a+b x)\right]^2 \tan\left[\frac{1}{2}(a+b x)\right] + \\
& 2 \left(-\frac{1}{2+m}m^2 \operatorname{AppellF1}\left[1 + \frac{m}{2}, 1 - m, 1 + 2m, 1 + \frac{2+m}{2}, \tan\left[\frac{1}{2}(a+b x)\right]^2, \right. \right. \\
& \quad \left.- \tan\left[\frac{1}{2}(a+b x)\right]^2\right] \sec\left[\frac{1}{2}(a+b x)\right]^2 \tan\left[\frac{1}{2}(a+b x)\right] - \frac{1}{2+m} \\
& m(1+2m) \operatorname{AppellF1}\left[1 + \frac{m}{2}, -m, 2+2m, 1 + \frac{2+m}{2}, \tan\left[\frac{1}{2}(a+b x)\right]^2, \right. \\
& \quad \left.- \tan\left[\frac{1}{2}(a+b x)\right]^2\right] \sec\left[\frac{1}{2}(a+b x)\right]^2 \tan\left[\frac{1}{2}(a+b x)\right]\Big)\Big)\Bigg) \\
& \left((-2+m) \left(-\operatorname{AppellF1}\left[\frac{1}{2}(-2+m), -m, 2m, \frac{m}{2}, \tan\left[\frac{1}{2}(a+b x)\right]^2, \right. \right. \right. \\
& \quad \left.- \tan\left[\frac{1}{2}(a+b x)\right]^2\right] + 2 \left(\operatorname{AppellF1}\left[\frac{m}{2}, 1-m, 2m, \frac{2+m}{2}, \tan\left[\frac{1}{2}(a+b x)\right]^2, \right. \right. \\
& \quad \left.- \tan\left[\frac{1}{2}(a+b x)\right]^2\right] + 2 \operatorname{AppellF1}\left[\frac{m}{2}, -m, 1+2m, \frac{2+m}{2}, \right. \\
& \quad \left.\tan\left[\frac{1}{2}(a+b x)\right]^2, -\tan\left[\frac{1}{2}(a+b x)\right]^2\right] \tan\left[\frac{1}{2}(a+b x)\right]^2\Big)^2\Big) - \\
& \left(2(2+m) \operatorname{AppellF1}\left[\frac{m}{2}, -m, 2m, \frac{2+m}{2}, \tan\left[\frac{1}{2}(a+b x)\right]^2, -\tan\left[\frac{1}{2}(a+b x)\right]^2\right] \right. \\
& \quad \left(-2m \left(\operatorname{AppellF1}\left[\frac{2+m}{2}, 1-m, 2m, \frac{4+m}{2}, \tan\left[\frac{1}{2}(a+b x)\right]^2, -\tan\left[\frac{1}{2}(a+b x)\right]^2\right] + \right. \right. \\
& \quad \left.2 \operatorname{AppellF1}\left[\frac{2+m}{2}, -m, 1+2m, \frac{4+m}{2}, \tan\left[\frac{1}{2}(a+b x)\right]^2, -\tan\left[\frac{1}{2}(a+b x)\right]^2\right]\right) \\
& \quad \sec\left[\frac{1}{2}(a+b x)\right]^2 \tan\left[\frac{1}{2}(a+b x)\right] + (2+m) \left(-\frac{1}{2+m}m^2 \operatorname{AppellF1}\left[1 + \frac{m}{2}, 1 - m, \right. \right. \\
& \quad \left.2m, 1 + \frac{2+m}{2}, \tan\left[\frac{1}{2}(a+b x)\right]^2, -\tan\left[\frac{1}{2}(a+b x)\right]^2\right] \sec\left[\frac{1}{2}(a+b x)\right]^2 \\
& \quad \tan\left[\frac{1}{2}(a+b x)\right] - \frac{1}{2+m}2m^2 \operatorname{AppellF1}\left[1 + \frac{m}{2}, -m, 1 + 2m, 1 + \frac{2+m}{2}, \right. \\
& \quad \left.\tan\left[\frac{1}{2}(a+b x)\right]^2, -\tan\left[\frac{1}{2}(a+b x)\right]^2\right] \sec\left[\frac{1}{2}(a+b x)\right]^2 \tan\left[\frac{1}{2}(a+b x)\right]\Big) - \\
& 2m \tan\left[\frac{1}{2}(a+b x)\right]^2 \left(-\frac{1}{4+m}2m(2+m) \operatorname{AppellF1}\left[1 + \frac{2+m}{2}, 1 - m, 1 + 2m, 1 + \frac{4+m}{2}, \right. \right. \\
& \quad \left.\tan\left[\frac{1}{2}(a+b x)\right]^2, -\tan\left[\frac{1}{2}(a+b x)\right]^2\right] \sec\left[\frac{1}{2}(a+b x)\right]^2 \tan\left[\frac{1}{2}(a+b x)\right] + \\
& \quad \frac{1}{4+m}(1-m)(2+m) \operatorname{AppellF1}\left[1 + \frac{2+m}{2}, 2 - m, 2m, 1 + \frac{4+m}{2}, \right. \\
& \quad \left.\tan\left[\frac{1}{2}(a+b x)\right]^2, -\tan\left[\frac{1}{2}(a+b x)\right]^2\right] \sec\left[\frac{1}{2}(a+b x)\right]^2 \tan\left[\frac{1}{2}(a+b x)\right] + \\
& 2 \left(-\frac{1}{4+m}m(2+m) \operatorname{AppellF1}\left[1 + \frac{2+m}{2}, 1 - m, 1 + 2m, 1 + \frac{4+m}{2}, \tan\left[\frac{1}{2}(a+b x)\right]^2, \right. \right. \\
& \quad \left.- \tan\left[\frac{1}{2}(a+b x)\right]^2\right] \sec\left[\frac{1}{2}(a+b x)\right]^2 \tan\left[\frac{1}{2}(a+b x)\right] - \frac{1}{4+m}(2+m)
\end{aligned}$$

$$\begin{aligned} & -\operatorname{Tan}\left[\frac{1}{2} (a+b x)\right]^2] - 2 m \left(\operatorname{AppellF1}\left[\frac{4+m}{2}, 1-m, 2m, \frac{6+m}{2}, \right. \right. \\ & \left. \left. \operatorname{Tan}\left[\frac{1}{2} (a+b x)\right]^2, -\operatorname{Tan}\left[\frac{1}{2} (a+b x)\right]^2\right] + 2 \operatorname{AppellF1}\left[\frac{4+m}{2}, -m, 1+2m, \right. \right. \\ & \left. \left. \frac{6+m}{2}, \operatorname{Tan}\left[\frac{1}{2} (a+b x)\right]^2, -\operatorname{Tan}\left[\frac{1}{2} (a+b x)\right]^2\right]\right) \operatorname{Tan}\left[\frac{1}{2} (a+b x)\right]^2\right)^2\right)\right) \end{aligned}$$

Problem 136: Result more than twice size of optimal antiderivative.

$$\int \cos[a+b x] \csc[2 a+2 b x] dx$$

Optimal (type 3, 14 leaves, 2 steps):

$$-\frac{\operatorname{ArcTanh}[\cos[a+b x]]}{2 b}$$

Result (type 3, 42 leaves):

$$\frac{1}{2} \left(-\frac{\operatorname{Log}[\cos[\frac{a}{2} + \frac{b x}{2}]]}{b} + \frac{\operatorname{Log}[\sin[\frac{a}{2} + \frac{b x}{2}]]}{b} \right)$$

Problem 137: Result more than twice size of optimal antiderivative.

$$\int \cos[a+b x] \csc[2 a+2 b x]^2 dx$$

Optimal (type 3, 28 leaves, 4 steps):

$$\frac{\operatorname{ArcTanh}[\sin[a+b x]]}{4 b} - \frac{\csc[a+b x]}{4 b}$$

Result (type 3, 94 leaves):

$$\begin{aligned} & \frac{1}{4} \left(-\frac{\cot[\frac{1}{2} (a+b x)]}{2 b} - \frac{\operatorname{Log}[\cos[\frac{1}{2} (a+b x)] - \sin[\frac{1}{2} (a+b x)]]}{b} + \right. \\ & \left. \frac{\operatorname{Log}[\cos[\frac{1}{2} (a+b x)] + \sin[\frac{1}{2} (a+b x)]]}{b} - \frac{\tan[\frac{1}{2} (a+b x)]}{2 b} \right) \end{aligned}$$

Problem 138: Result more than twice size of optimal antiderivative.

$$\int \cos[a+b x] \csc[2 a+2 b x]^3 dx$$

Optimal (type 3, 49 leaves, 5 steps):

$$-\frac{3 \operatorname{ArcTanh}[\cos[a+b x]]}{16 b} + \frac{3 \sec[a+b x]}{16 b} - \frac{\csc[a+b x]^2 \sec[a+b x]}{16 b}$$

Result (type 3, 143 leaves) :

$$\begin{aligned} & \left(\csc[a + bx]^4 \left(2 - 6 \cos[2(a + bx)] + 2 \cos[3(a + bx)] \right) + \right. \\ & \quad 3 \cos[3(a + bx)] \log[\cos[\frac{1}{2}(a + bx)]] - 3 \cos[3(a + bx)] \log[\sin[\frac{1}{2}(a + bx)]] + \\ & \quad \cos[a + bx] \left(-2 - 3 \log[\cos[\frac{1}{2}(a + bx)]] + 3 \log[\sin[\frac{1}{2}(a + bx)]] \right) \Big) \Big) / \\ & \left(16b \left(\csc[\frac{1}{2}(a + bx)]^2 - \sec[\frac{1}{2}(a + bx)]^2 \right) \right) \end{aligned}$$

Problem 139: Result more than twice size of optimal antiderivative.

$$\int \cos[a + bx] \csc[2a + 2bx]^4 dx$$

Optimal (type 3, 66 leaves, 6 steps) :

$$\frac{5 \operatorname{ArcTanh}[\sin[a + bx]]}{32b} - \frac{5 \csc[a + bx]}{32b} - \frac{5 \csc[a + bx]^3}{96b} + \frac{\csc[a + bx]^3 \sec[a + bx]^2}{32b}$$

Result (type 3, 215 leaves) :

$$\begin{aligned} & -\frac{13 \cot[\frac{1}{2}(a + bx)]}{192b} - \frac{\cot[\frac{1}{2}(a + bx)] \csc[\frac{1}{2}(a + bx)]^2}{384b} - \\ & \frac{5 \log[\cos[\frac{1}{2}(a + bx)] - \sin[\frac{1}{2}(a + bx)]]}{32b} + \frac{5 \log[\cos[\frac{1}{2}(a + bx)] + \sin[\frac{1}{2}(a + bx)]]}{32b} + \\ & \frac{1}{64b (\cos[\frac{1}{2}(a + bx)] - \sin[\frac{1}{2}(a + bx)])^2} - \frac{1}{64b (\cos[\frac{1}{2}(a + bx)] + \sin[\frac{1}{2}(a + bx)])^2} - \\ & \frac{13 \tan[\frac{1}{2}(a + bx)]}{192b} - \frac{\sec[\frac{1}{2}(a + bx)]^2 \tan[\frac{1}{2}(a + bx)]}{384b} \end{aligned}$$

Problem 140: Result more than twice size of optimal antiderivative.

$$\int \cos[a + bx] \csc[2a + 2bx]^5 dx$$

Optimal (type 3, 89 leaves, 7 steps) :

$$\begin{aligned} & -\frac{35 \operatorname{ArcTanh}[\cos[a + bx]]}{256b} + \frac{35 \sec[a + bx]}{256b} + \\ & \frac{35 \sec[a + bx]^3}{768b} - \frac{7 \csc[a + bx]^2 \sec[a + bx]^3}{256b} - \frac{\csc[a + bx]^4 \sec[a + bx]^3}{128b} \end{aligned}$$

Result (type 3, 268 leaves) :

$$\begin{aligned}
 & -\frac{1}{768 b \left(\csc\left[\frac{1}{2} (a+b x)\right]^2 - \sec\left[\frac{1}{2} (a+b x)\right]^2\right)^3} \csc[a+b x]^{10} \\
 & \left(-204 + 658 \cos[2 (a+b x)] - 228 \cos[3 (a+b x)] + 140 \cos[4 (a+b x)] - 76 \cos[5 (a+b x)] - \right. \\
 & 210 \cos[6 (a+b x)] + 76 \cos[7 (a+b x)] - 315 \cos[3 (a+b x)] \log[\cos[\frac{1}{2} (a+b x)]] - \\
 & 105 \cos[5 (a+b x)] \log[\cos[\frac{1}{2} (a+b x)]] + 105 \cos[7 (a+b x)] \log[\cos[\frac{1}{2} (a+b x)]] + \\
 & 3 \cos[a+b x] \left(76 + 105 \log[\cos[\frac{1}{2} (a+b x)]] - 105 \log[\sin[\frac{1}{2} (a+b x)]] \right) + \\
 & 315 \cos[3 (a+b x)] \log[\sin[\frac{1}{2} (a+b x)]] + \\
 & \left. 105 \cos[5 (a+b x)] \log[\sin[\frac{1}{2} (a+b x)]] - 105 \cos[7 (a+b x)] \log[\sin[\frac{1}{2} (a+b x)]] \right)
 \end{aligned}$$

Problem 158: Result more than twice size of optimal antiderivative.

$$\int \cos[a+b x]^3 \csc[2 a+2 b x]^3 dx$$

Optimal (type 3, 34 leaves, 3 steps):

$$\begin{aligned}
 & -\frac{\text{ArcTanh}[\cos[a+b x]]}{16 b} - \frac{\cot[a+b x] \csc[a+b x]}{16 b}
 \end{aligned}$$

Result (type 3, 79 leaves):

$$\frac{1}{8} \left(-\frac{\csc[\frac{1}{2} (a+b x)]^2}{8 b} - \frac{\log[\cos[\frac{1}{2} (a+b x)]]}{2 b} + \frac{\log[\sin[\frac{1}{2} (a+b x)]]}{2 b} + \frac{\sec[\frac{1}{2} (a+b x)]^2}{8 b} \right)$$

Problem 159: Result more than twice size of optimal antiderivative.

$$\int \cos[a+b x]^3 \csc[2 a+2 b x]^4 dx$$

Optimal (type 3, 43 leaves, 5 steps):

$$\begin{aligned}
 & \frac{\text{ArcTanh}[\sin[a+b x]]}{16 b} - \frac{\csc[a+b x]}{16 b} - \frac{\csc[a+b x]^3}{48 b}
 \end{aligned}$$

Result (type 3, 152 leaves):

$$\frac{1}{16} \left(-\frac{7 \operatorname{Cot}\left[\frac{1}{2} (a + b x)\right]}{12 b} - \frac{\operatorname{Cot}\left[\frac{1}{2} (a + b x)\right] \csc^2\left[\frac{1}{2} (a + b x)\right]}{24 b} - \frac{\operatorname{Log}[\cos[\frac{1}{2} (a + b x)] - \sin[\frac{1}{2} (a + b x)]]}{b} + \frac{\operatorname{Log}[\cos[\frac{1}{2} (a + b x)] + \sin[\frac{1}{2} (a + b x)]]}{b} - \frac{7 \tan[\frac{1}{2} (a + b x)]}{12 b} - \frac{\sec^2[\frac{1}{2} (a + b x)] \tan[\frac{1}{2} (a + b x)]}{24 b} \right)$$

Problem 160: Result more than twice size of optimal antiderivative.

$$\int \cos[a + b x]^3 \csc[2 a + 2 b x]^5 dx$$

Optimal (type 3, 70 leaves, 6 steps):

$$-\frac{15 \operatorname{ArcTanh}[\cos[a + b x]]}{256 b} + \frac{15 \sec[a + b x]}{256 b} - \frac{5 \csc[a + b x]^2 \sec[a + b x]}{256 b} - \frac{\csc[a + b x]^4 \sec[a + b x]}{128 b}$$

Result (type 3, 195 leaves):

$$-\frac{7 \csc^2[\frac{1}{2} (a + b x)]}{1024 b} - \frac{\csc^4[\frac{1}{2} (a + b x)]}{2048 b} - \frac{15 \operatorname{Log}[\cos[\frac{1}{2} (a + b x)]]}{256 b} + \frac{15 \operatorname{Log}[\sin[\frac{1}{2} (a + b x)]]}{256 b} + \frac{7 \sec^2[\frac{1}{2} (a + b x)]^2}{1024 b} + \frac{\sec^4[\frac{1}{2} (a + b x)]}{2048 b} + \frac{\sin[\frac{1}{2} (a + b x)]}{32 b (\cos[\frac{1}{2} (a + b x)] - \sin[\frac{1}{2} (a + b x)])} - \frac{\sin[\frac{1}{2} (a + b x)]}{32 b (\cos[\frac{1}{2} (a + b x)] + \sin[\frac{1}{2} (a + b x)])}$$

Problem 187: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \cos[a + b x]^3 \sin[2 a + 2 b x]^m dx$$

Optimal (type 5, 85 leaves, 2 steps):

$$-\frac{1}{b (4 + m)} \cos[a + b x]^3 \cot[a + b x] \operatorname{Hypergeometric2F1}\left[\frac{1-m}{2}, \frac{4+m}{2}, \frac{6+m}{2}, \cos[a + b x]^2\right] (\sin[a + b x]^2)^{\frac{1-m}{2}} \sin[2 a + 2 b x]^m$$

Result (type 6, 10498 leaves):

$$-\left(\left(2^{1+2m} (3+m) \cos[a + b x]^3 \right. \right.$$

$$\begin{aligned}
& \text{Sin}[2(a + bx)]^m \tan\left[\frac{1}{2}(a + bx)\right] \left(\frac{\tan\left[\frac{1}{2}(a + bx)\right] - \tan\left[\frac{1}{2}(a + bx)\right]^3}{\left(1 + \tan\left[\frac{1}{2}(a + bx)\right]^2\right)^2} \right)^m \\
& \left(\left(\text{AppellF1}\left[\frac{1+m}{2}, -m, 1+2m, \frac{3+m}{2}, \tan\left[\frac{1}{2}(a + bx)\right]^2, -\tan\left[\frac{1}{2}(a + bx)\right]^2 \right] \right. \right. \\
& \left. \left. \left(1 + \tan\left[\frac{1}{2}(a + bx)\right]^2 \right)^3 \right) / \right. \\
& \left((3+m) \text{AppellF1}\left[\frac{1+m}{2}, -m, 1+2m, \frac{3+m}{2}, \tan\left[\frac{1}{2}(a + bx)\right]^2, -\tan\left[\frac{1}{2}(a + bx)\right]^2 \right] - \right. \\
& \left. 2 \left(m \text{AppellF1}\left[\frac{3+m}{2}, 1-m, 1+2m, \frac{5+m}{2}, \tan\left[\frac{1}{2}(a + bx)\right]^2, -\tan\left[\frac{1}{2}(a + bx)\right]^2 \right] + \right. \right. \\
& \left. \left. (1+2m) \text{AppellF1}\left[\frac{3+m}{2}, -m, 2(1+m), \frac{5+m}{2}, \right. \right. \\
& \left. \left. \tan\left[\frac{1}{2}(a + bx)\right]^2, -\tan\left[\frac{1}{2}(a + bx)\right]^2 \right] \right) \tan\left[\frac{1}{2}(a + bx)\right]^2 \right) + \\
& \left(12 \text{AppellF1}\left[\frac{1+m}{2}, -m, 3+2m, \frac{3+m}{2}, \tan\left[\frac{1}{2}(a + bx)\right]^2, -\tan\left[\frac{1}{2}(a + bx)\right]^2 \right] \right. \\
& \left. \left(1 + \tan\left[\frac{1}{2}(a + bx)\right]^2 \right) \right) / \\
& \left((3+m) \text{AppellF1}\left[\frac{1+m}{2}, -m, 3+2m, \frac{3+m}{2}, \tan\left[\frac{1}{2}(a + bx)\right]^2, -\tan\left[\frac{1}{2}(a + bx)\right]^2 \right] - \right. \\
& \left. 2 \left(m \text{AppellF1}\left[\frac{3+m}{2}, 1-m, 3+2m, \frac{5+m}{2}, \tan\left[\frac{1}{2}(a + bx)\right]^2, -\tan\left[\frac{1}{2}(a + bx)\right]^2 \right] + \right. \right. \\
& \left. \left. (3+2m) \text{AppellF1}\left[\frac{3+m}{2}, -m, 2(2+m), \frac{5+m}{2}, \right. \right. \\
& \left. \left. \tan\left[\frac{1}{2}(a + bx)\right]^2, -\tan\left[\frac{1}{2}(a + bx)\right]^2 \right] \right) \tan\left[\frac{1}{2}(a + bx)\right]^2 \right) - \\
& \left(6 \text{AppellF1}\left[\frac{1+m}{2}, -m, 2(1+m), \frac{3+m}{2}, \tan\left[\frac{1}{2}(a + bx)\right]^2, -\tan\left[\frac{1}{2}(a + bx)\right]^2 \right] \right. \\
& \left. \left(1 + \tan\left[\frac{1}{2}(a + bx)\right]^2 \right)^2 \right) / \\
& \left((3+m) \text{AppellF1}\left[\frac{1+m}{2}, -m, 2(1+m), \frac{3+m}{2}, \tan\left[\frac{1}{2}(a + bx)\right]^2, -\tan\left[\frac{1}{2}(a + bx)\right]^2 \right] - \right. \\
& \left. 2 \left(m \text{AppellF1}\left[\frac{3+m}{2}, 1-m, 2(1+m), \frac{5+m}{2}, \tan\left[\frac{1}{2}(a + bx)\right]^2, -\tan\left[\frac{1}{2}(a + bx)\right]^2 \right] + \right. \right. \\
& \left. \left. 2(1+m) \text{AppellF1}\left[\frac{3+m}{2}, -m, 3+2m, \frac{5+m}{2}, \right. \right. \\
& \left. \left. \tan\left[\frac{1}{2}(a + bx)\right]^2, -\tan\left[\frac{1}{2}(a + bx)\right]^2 \right] \right) \tan\left[\frac{1}{2}(a + bx)\right]^2 \right) - \\
& \left(8 \text{AppellF1}\left[\frac{1+m}{2}, -m, 2(2+m), \frac{3+m}{2}, \tan\left[\frac{1}{2}(a + bx)\right]^2, -\tan\left[\frac{1}{2}(a + bx)\right]^2 \right] \right) / \\
& \left((3+m) \text{AppellF1}\left[\frac{1+m}{2}, -m, 2(2+m), \frac{3+m}{2}, \tan\left[\frac{1}{2}(a + bx)\right]^2, -\tan\left[\frac{1}{2}(a + bx)\right]^2 \right] - \right. \\
& \left. 2 \left(m \text{AppellF1}\left[\frac{3+m}{2}, 1-m, 2(2+m), \frac{5+m}{2}, \tan\left[\frac{1}{2}(a + bx)\right]^2, -\tan\left[\frac{1}{2}(a + bx)\right]^2 \right] + \right. \right.
\end{aligned}$$

$$\begin{aligned}
& \left. 2 \left(2 + m \right) \text{AppellF1} \left[\frac{3+m}{2}, -m, 5+2m, \frac{5+m}{2}, \tan \left[\frac{1}{2} (a+b x) \right]^2, \right. \right. \\
& \quad \left. \left. -\tan \left[\frac{1}{2} (a+b x) \right]^2 \right] \tan \left[\frac{1}{2} (a+b x) \right]^2 \right) \Bigg) \Bigg) \Bigg/ \\
& \left(b (1+m) \left(1 + \tan \left[\frac{1}{2} (a+b x) \right]^2 \right)^4 \left(\frac{1}{(1+m) \left(1 + \tan \left[\frac{1}{2} (a+b x) \right]^2 \right)^5} \right. \right. \\
& \quad \left. \left. 2^{3+2m} (3+m) \sec \left[\frac{1}{2} (a+b x) \right]^2 \tan \left[\frac{1}{2} (a+b x) \right]^2 \right. \right. \\
& \quad \left. \left. \left(\frac{\tan \left[\frac{1}{2} (a+b x) \right] - \tan \left[\frac{1}{2} (a+b x) \right]^3}{\left(1 + \tan \left[\frac{1}{2} (a+b x) \right]^2 \right)^2} \right)^m \left(\left(\text{AppellF1} \left[\frac{1+m}{2}, -m, 1+2m, \frac{3+m}{2}, \right. \right. \right. \right. \right. \\
& \quad \left. \left. \left. \left. \left. \tan \left[\frac{1}{2} (a+b x) \right]^2, -\tan \left[\frac{1}{2} (a+b x) \right]^2 \right] \left(1 + \tan \left[\frac{1}{2} (a+b x) \right]^2 \right)^3 \right) \Bigg) \Bigg/ \right. \\
& \quad \left((3+m) \text{AppellF1} \left[\frac{1+m}{2}, -m, 1+2m, \frac{3+m}{2}, \tan \left[\frac{1}{2} (a+b x) \right]^2, -\tan \left[\frac{1}{2} (a+b x) \right]^2 \right] - \right. \\
& \quad \left. 2 \left(m \text{AppellF1} \left[\frac{3+m}{2}, 1-m, 1+2m, \frac{5+m}{2}, \tan \left[\frac{1}{2} (a+b x) \right]^2, -\tan \left[\frac{1}{2} (a+b x) \right]^2 \right] + \right. \right. \\
& \quad \left. \left. (1+2m) \text{AppellF1} \left[\frac{3+m}{2}, -m, 2(1+m), \frac{5+m}{2}, \tan \left[\frac{1}{2} (a+b x) \right]^2, \right. \right. \\
& \quad \left. \left. -\tan \left[\frac{1}{2} (a+b x) \right]^2 \right] \tan \left[\frac{1}{2} (a+b x) \right]^2 \right) + \left(12 \text{AppellF1} \left[\frac{1+m}{2}, -m, 3+2m, \right. \right. \\
& \quad \left. \left. \frac{3+m}{2}, \tan \left[\frac{1}{2} (a+b x) \right]^2, -\tan \left[\frac{1}{2} (a+b x) \right]^2 \right] \left(1 + \tan \left[\frac{1}{2} (a+b x) \right]^2 \right) \Bigg) \Bigg/ \right. \\
& \quad \left((3+m) \text{AppellF1} \left[\frac{1+m}{2}, -m, 3+2m, \frac{3+m}{2}, \tan \left[\frac{1}{2} (a+b x) \right]^2, -\tan \left[\frac{1}{2} (a+b x) \right]^2 \right] - \right. \\
& \quad \left. 2 \left(m \text{AppellF1} \left[\frac{3+m}{2}, 1-m, 3+2m, \frac{5+m}{2}, \tan \left[\frac{1}{2} (a+b x) \right]^2, -\tan \left[\frac{1}{2} (a+b x) \right]^2 \right] + \right. \right. \\
& \quad \left. \left. (3+2m) \text{AppellF1} \left[\frac{3+m}{2}, -m, 2(2+m), \frac{5+m}{2}, \tan \left[\frac{1}{2} (a+b x) \right]^2, \right. \right. \\
& \quad \left. \left. -\tan \left[\frac{1}{2} (a+b x) \right]^2 \right] \tan \left[\frac{1}{2} (a+b x) \right]^2 \right) - \left(6 \text{AppellF1} \left[\frac{1+m}{2}, -m, 2(1+m), \right. \right. \\
& \quad \left. \left. \frac{3+m}{2}, \tan \left[\frac{1}{2} (a+b x) \right]^2, -\tan \left[\frac{1}{2} (a+b x) \right]^2 \right] \left(1 + \tan \left[\frac{1}{2} (a+b x) \right]^2 \right)^2 \right) \Bigg/ \right. \\
& \quad \left((3+m) \text{AppellF1} \left[\frac{1+m}{2}, -m, 2(1+m), \frac{3+m}{2}, \tan \left[\frac{1}{2} (a+b x) \right]^2, \right. \right. \\
& \quad \left. \left. -\tan \left[\frac{1}{2} (a+b x) \right]^2 \right] - 2 \left(m \text{AppellF1} \left[\frac{3+m}{2}, 1-m, 2(1+m), \frac{5+m}{2}, \right. \right. \right. \\
& \quad \left. \left. \left. \tan \left[\frac{1}{2} (a+b x) \right]^2, -\tan \left[\frac{1}{2} (a+b x) \right]^2 \right] + 2(1+m) \text{AppellF1} \left[\frac{3+m}{2}, -m, \right. \right. \\
& \quad \left. \left. 3+2m, \frac{5+m}{2}, \tan \left[\frac{1}{2} (a+b x) \right]^2, -\tan \left[\frac{1}{2} (a+b x) \right]^2 \right] \tan \left[\frac{1}{2} (a+b x) \right]^2 \right) - \right. \\
& \quad \left(8 \text{AppellF1} \left[\frac{1+m}{2}, -m, 2(2+m), \frac{3+m}{2}, \tan \left[\frac{1}{2} (a+b x) \right]^2, -\tan \left[\frac{1}{2} (a+b x) \right]^2 \right] \right) \Bigg/
\end{aligned}$$

$$\begin{aligned}
& \left((3+m) \operatorname{AppellF1} \left[\frac{1+m}{2}, -m, 2(2+m), \frac{3+m}{2}, \tan \left[\frac{1}{2}(a+b x) \right]^2, \right. \right. \\
& \quad \left. \left. -\tan \left[\frac{1}{2}(a+b x) \right]^2 \right] - 2 \left(m \operatorname{AppellF1} \left[\frac{3+m}{2}, 1-m, 2(2+m), \frac{5+m}{2}, \right. \right. \\
& \quad \left. \left. \tan \left[\frac{1}{2}(a+b x) \right]^2, -\tan \left[\frac{1}{2}(a+b x) \right]^2 \right] + 2(2+m) \operatorname{AppellF1} \left[\frac{3+m}{2}, -m, \right. \right. \\
& \quad \left. \left. 5+2m, \frac{5+m}{2}, \tan \left[\frac{1}{2}(a+b x) \right]^2, -\tan \left[\frac{1}{2}(a+b x) \right]^2 \right] \right) \tan \left[\frac{1}{2}(a+b x) \right]^2 \right) - \\
& \frac{1}{(1+m) \left(1 + \tan \left[\frac{1}{2}(a+b x) \right]^2 \right)^4} 2^{2m} (3+m) \sec \left[\frac{1}{2}(a+b x) \right]^2 \\
& \left(\frac{\tan \left[\frac{1}{2}(a+b x) \right] - \tan \left[\frac{1}{2}(a+b x) \right]^3}{\left(1 + \tan \left[\frac{1}{2}(a+b x) \right]^2 \right)^2} \right)^m \\
& \left(\left(\operatorname{AppellF1} \left[\frac{1+m}{2}, -m, 1+2m, \frac{3+m}{2}, \tan \left[\frac{1}{2}(a+b x) \right]^2, \right. \right. \right. \\
& \quad \left. \left. \left. -\tan \left[\frac{1}{2}(a+b x) \right]^2 \right] \left(1 + \tan \left[\frac{1}{2}(a+b x) \right]^2 \right)^3 \right) / \\
& \left((3+m) \operatorname{AppellF1} \left[\frac{1+m}{2}, -m, 1+2m, \frac{3+m}{2}, \tan \left[\frac{1}{2}(a+b x) \right]^2, -\tan \left[\frac{1}{2}(a+b x) \right]^2 \right] - \right. \\
& \quad \left. 2 \left(m \operatorname{AppellF1} \left[\frac{3+m}{2}, 1-m, 1+2m, \frac{5+m}{2}, \tan \left[\frac{1}{2}(a+b x) \right]^2, -\tan \left[\frac{1}{2}(a+b x) \right]^2 \right] + \right. \right. \\
& \quad \left. \left. (1+2m) \operatorname{AppellF1} \left[\frac{3+m}{2}, -m, 2(1+m), \frac{5+m}{2}, \tan \left[\frac{1}{2}(a+b x) \right]^2, \right. \right. \\
& \quad \left. \left. -\tan \left[\frac{1}{2}(a+b x) \right]^2 \right] \tan \left[\frac{1}{2}(a+b x) \right]^2 \right) + \left(12 \operatorname{AppellF1} \left[\frac{1+m}{2}, -m, 3+2m, \right. \right. \\
& \quad \left. \left. \frac{3+m}{2}, \tan \left[\frac{1}{2}(a+b x) \right]^2, -\tan \left[\frac{1}{2}(a+b x) \right]^2 \right] \left(1 + \tan \left[\frac{1}{2}(a+b x) \right]^2 \right) \right) / \\
& \left((3+m) \operatorname{AppellF1} \left[\frac{1+m}{2}, -m, 3+2m, \frac{3+m}{2}, \tan \left[\frac{1}{2}(a+b x) \right]^2, -\tan \left[\frac{1}{2}(a+b x) \right]^2 \right] - \right. \\
& \quad \left. 2 \left(m \operatorname{AppellF1} \left[\frac{3+m}{2}, 1-m, 3+2m, \frac{5+m}{2}, \tan \left[\frac{1}{2}(a+b x) \right]^2, -\tan \left[\frac{1}{2}(a+b x) \right]^2 \right] + \right. \right. \\
& \quad \left. \left. (3+2m) \operatorname{AppellF1} \left[\frac{3+m}{2}, -m, 2(2+m), \frac{5+m}{2}, \tan \left[\frac{1}{2}(a+b x) \right]^2, \right. \right. \\
& \quad \left. \left. -\tan \left[\frac{1}{2}(a+b x) \right]^2 \right] \tan \left[\frac{1}{2}(a+b x) \right]^2 \right) - \left(6 \operatorname{AppellF1} \left[\frac{1+m}{2}, -m, 2(1+m), \right. \right. \\
& \quad \left. \left. \frac{3+m}{2}, \tan \left[\frac{1}{2}(a+b x) \right]^2, -\tan \left[\frac{1}{2}(a+b x) \right]^2 \right] \left(1 + \tan \left[\frac{1}{2}(a+b x) \right]^2 \right)^2 \right) / \\
& \left((3+m) \operatorname{AppellF1} \left[\frac{1+m}{2}, -m, 2(1+m), \frac{3+m}{2}, \tan \left[\frac{1}{2}(a+b x) \right]^2, \right. \right. \\
& \quad \left. \left. -\tan \left[\frac{1}{2}(a+b x) \right]^2 \right] - 2 \left(m \operatorname{AppellF1} \left[\frac{3+m}{2}, 1-m, 2(1+m), \frac{5+m}{2}, \right. \right. \\
& \quad \left. \left. \tan \left[\frac{1}{2}(a+b x) \right]^2, -\tan \left[\frac{1}{2}(a+b x) \right]^2 \right] + 2(1+m) \operatorname{AppellF1} \left[\frac{3+m}{2}, -m, \right. \right. \\
& \quad \left. \left. 3+2m, \frac{5+m}{2}, \tan \left[\frac{1}{2}(a+b x) \right]^2, -\tan \left[\frac{1}{2}(a+b x) \right]^2 \right] \right) \tan \left[\frac{1}{2}(a+b x) \right]^2 \right) -
\end{aligned}$$

$$\begin{aligned}
& \left(8 \operatorname{AppellF1} \left[\frac{1+m}{2}, -m, 2(2+m), \frac{3+m}{2}, \tan \left[\frac{1}{2}(a+b x) \right]^2, -\tan \left[\frac{1}{2}(a+b x) \right]^2 \right] \right) / \\
& \left((3+m) \operatorname{AppellF1} \left[\frac{1+m}{2}, -m, 2(2+m), \frac{3+m}{2}, \tan \left[\frac{1}{2}(a+b x) \right]^2, \right. \right. \\
& \left. \left. -\tan \left[\frac{1}{2}(a+b x) \right]^2 \right] - 2 \left(m \operatorname{AppellF1} \left[\frac{3+m}{2}, 1-m, 2(2+m), \frac{5+m}{2}, \right. \right. \\
& \left. \left. \tan \left[\frac{1}{2}(a+b x) \right]^2, -\tan \left[\frac{1}{2}(a+b x) \right]^2 \right] + 2(2+m) \operatorname{AppellF1} \left[\frac{3+m}{2}, -m, \right. \right. \\
& \left. \left. 5+2m, \frac{5+m}{2}, \tan \left[\frac{1}{2}(a+b x) \right]^2, -\tan \left[\frac{1}{2}(a+b x) \right]^2 \right] \right) \tan \left[\frac{1}{2}(a+b x) \right]^2 \right) - \\
& \frac{1}{(1+m) \left(1 + \tan \left[\frac{1}{2}(a+b x) \right]^2 \right)^4} 2^{1+2m} m (3+m) \tan \left[\frac{1}{2}(a+b x) \right] \\
& \left(\frac{\tan \left[\frac{1}{2}(a+b x) \right] - \tan \left[\frac{1}{2}(a+b x) \right]^3}{\left(1 + \tan \left[\frac{1}{2}(a+b x) \right]^2 \right)^2} \right)^{-1+m} \\
& \left(\left(\operatorname{AppellF1} \left[\frac{1+m}{2}, -m, 1+2m, \frac{3+m}{2}, \tan \left[\frac{1}{2}(a+b x) \right]^2, \right. \right. \right. \\
& \left. \left. \left. -\tan \left[\frac{1}{2}(a+b x) \right]^2 \right] \left(1 + \tan \left[\frac{1}{2}(a+b x) \right]^2 \right)^3 \right) / \\
& \left((3+m) \operatorname{AppellF1} \left[\frac{1+m}{2}, -m, 1+2m, \frac{3+m}{2}, \tan \left[\frac{1}{2}(a+b x) \right]^2, -\tan \left[\frac{1}{2}(a+b x) \right]^2 \right] - \right. \\
& \left. 2 \left(m \operatorname{AppellF1} \left[\frac{3+m}{2}, 1-m, 1+2m, \frac{5+m}{2}, \tan \left[\frac{1}{2}(a+b x) \right]^2, -\tan \left[\frac{1}{2}(a+b x) \right]^2 \right] + \right. \right. \\
& \left. \left. (1+2m) \operatorname{AppellF1} \left[\frac{3+m}{2}, -m, 2(1+m), \frac{5+m}{2}, \tan \left[\frac{1}{2}(a+b x) \right]^2, \right. \right. \\
& \left. \left. -\tan \left[\frac{1}{2}(a+b x) \right]^2 \right] \tan \left[\frac{1}{2}(a+b x) \right]^2 \right) + \left(12 \operatorname{AppellF1} \left[\frac{1+m}{2}, -m, 3+2m, \right. \right. \\
& \left. \left. \frac{3+m}{2}, \tan \left[\frac{1}{2}(a+b x) \right]^2, -\tan \left[\frac{1}{2}(a+b x) \right]^2 \right] \left(1 + \tan \left[\frac{1}{2}(a+b x) \right]^2 \right) \right) / \\
& \left((3+m) \operatorname{AppellF1} \left[\frac{1+m}{2}, -m, 3+2m, \frac{3+m}{2}, \tan \left[\frac{1}{2}(a+b x) \right]^2, -\tan \left[\frac{1}{2}(a+b x) \right]^2 \right] - \right. \\
& \left. 2 \left(m \operatorname{AppellF1} \left[\frac{3+m}{2}, 1-m, 3+2m, \frac{5+m}{2}, \tan \left[\frac{1}{2}(a+b x) \right]^2, -\tan \left[\frac{1}{2}(a+b x) \right]^2 \right] + \right. \right. \\
& \left. \left. (3+2m) \operatorname{AppellF1} \left[\frac{3+m}{2}, -m, 2(2+m), \frac{5+m}{2}, \tan \left[\frac{1}{2}(a+b x) \right]^2, \right. \right. \\
& \left. \left. -\tan \left[\frac{1}{2}(a+b x) \right]^2 \right] \tan \left[\frac{1}{2}(a+b x) \right]^2 \right) - \left(6 \operatorname{AppellF1} \left[\frac{1+m}{2}, -m, 2(1+m), \right. \right. \\
& \left. \left. \frac{3+m}{2}, \tan \left[\frac{1}{2}(a+b x) \right]^2, -\tan \left[\frac{1}{2}(a+b x) \right]^2 \right] \left(1 + \tan \left[\frac{1}{2}(a+b x) \right]^2 \right)^2 \right) / \\
& \left((3+m) \operatorname{AppellF1} \left[\frac{1+m}{2}, -m, 2(1+m), \frac{3+m}{2}, \tan \left[\frac{1}{2}(a+b x) \right]^2, \right. \right. \\
& \left. \left. -\tan \left[\frac{1}{2}(a+b x) \right]^2 \right] - 2 \left(m \operatorname{AppellF1} \left[\frac{3+m}{2}, 1-m, 2(1+m), \frac{5+m}{2}, \right. \right. \\
& \left. \left. \tan \left[\frac{1}{2}(a+b x) \right]^2, -\tan \left[\frac{1}{2}(a+b x) \right]^2 \right] + 2(1+m) \operatorname{AppellF1} \left[\frac{3+m}{2}, -m, \right. \right. \\
& \left. \left. \frac{3+m}{2}, \tan \left[\frac{1}{2}(a+b x) \right]^2, -\tan \left[\frac{1}{2}(a+b x) \right]^2 \right] \right)
\end{aligned}$$

$$\begin{aligned}
& \left. 3 + 2m, \frac{5+m}{2}, \tan\left[\frac{1}{2}(a+b x)\right]^2, -\tan\left[\frac{1}{2}(a+b x)\right]^2\right) \tan\left[\frac{1}{2}(a+b x)\right]^2\Big) - \\
& \left(8 \operatorname{AppellF1}\left[\frac{1+m}{2}, -m, 2(2+m), \frac{3+m}{2}, \tan\left[\frac{1}{2}(a+b x)\right]^2, -\tan\left[\frac{1}{2}(a+b x)\right]^2\right]\right) / \\
& \left(\left(3+m\right) \operatorname{AppellF1}\left[\frac{1+m}{2}, -m, 2(2+m), \frac{3+m}{2}, \tan\left[\frac{1}{2}(a+b x)\right]^2,\right.\right. \\
& \left.-\tan\left[\frac{1}{2}(a+b x)\right]^2\right] - 2 \left(m \operatorname{AppellF1}\left[\frac{3+m}{2}, 1-m, 2(2+m), \frac{5+m}{2},\right.\right. \\
& \left.\left.\tan\left[\frac{1}{2}(a+b x)\right]^2, -\tan\left[\frac{1}{2}(a+b x)\right]^2\right] + 2(2+m) \operatorname{AppellF1}\left[\frac{3+m}{2}, -m,\right.\right. \\
& \left.\left.5+2m, \frac{5+m}{2}, \tan\left[\frac{1}{2}(a+b x)\right]^2, -\tan\left[\frac{1}{2}(a+b x)\right]^2\right]\right) \tan\left[\frac{1}{2}(a+b x)\right]^2\Big) \\
& \left(\frac{\frac{1}{2} \sec\left[\frac{1}{2}(a+b x)\right]^2 - \frac{3}{2} \sec\left[\frac{1}{2}(a+b x)\right]^2 \tan\left[\frac{1}{2}(a+b x)\right]^2}{\left(1+\tan\left[\frac{1}{2}(a+b x)\right]^2\right)^2} - \right. \\
& \left.\left(2 \sec\left[\frac{1}{2}(a+b x)\right]^2 \tan\left[\frac{1}{2}(a+b x)\right] \left(\tan\left[\frac{1}{2}(a+b x)\right] - \tan\left[\frac{1}{2}(a+b x)\right]^3\right)\right)\right) / \\
& \left.\left(1+\tan\left[\frac{1}{2}(a+b x)\right]^2\right)^3\right) - \\
& \frac{1}{(1+m) \left(1+\tan\left[\frac{1}{2}(a+b x)\right]^2\right)^4} 2^{1+2m} (3+m) \tan\left[\frac{1}{2}(a+b x)\right] \\
& \left(\frac{\tan\left[\frac{1}{2}(a+b x)\right] - \tan\left[\frac{1}{2}(a+b x)\right]^3}{\left(1+\tan\left[\frac{1}{2}(a+b x)\right]^2\right)^2}\right)^m \\
& \left(\left(3 \operatorname{AppellF1}\left[\frac{1+m}{2}, -m, 1+2m, \frac{3+m}{2}, \tan\left[\frac{1}{2}(a+b x)\right]^2, -\tan\left[\frac{1}{2}(a+b x)\right]^2\right]\right.\right. \\
& \left.\left.\sec\left[\frac{1}{2}(a+b x)\right]^2 \tan\left[\frac{1}{2}(a+b x)\right] \left(1+\tan\left[\frac{1}{2}(a+b x)\right]^2\right)^2\right)\right) / \\
& \left(\left(3+m\right) \operatorname{AppellF1}\left[\frac{1+m}{2}, -m, 1+2m, \frac{3+m}{2}, \tan\left[\frac{1}{2}(a+b x)\right]^2, -\tan\left[\frac{1}{2}(a+b x)\right]^2\right] - \right. \\
& \left.2 \left(m \operatorname{AppellF1}\left[\frac{3+m}{2}, 1-m, 1+2m, \frac{5+m}{2}, \tan\left[\frac{1}{2}(a+b x)\right]^2, -\tan\left[\frac{1}{2}(a+b x)\right]^2\right] + \right.\right. \\
& \left.\left.(1+2m) \operatorname{AppellF1}\left[\frac{3+m}{2}, -m, 2(1+m), \frac{5+m}{2},\right.\right. \\
& \left.\left.\tan\left[\frac{1}{2}(a+b x)\right]^2, -\tan\left[\frac{1}{2}(a+b x)\right]^2\right]\right) \tan\left[\frac{1}{2}(a+b x)\right]^2\right) + \\
& \left(\left(-\frac{1}{3+m} m (1+m) \operatorname{AppellF1}\left[1+\frac{1+m}{2}, 1-m, 1+2m, 1+\frac{3+m}{2}, \tan\left[\frac{1}{2}(a+b x)\right]^2,\right.\right.\right. \\
& \left.\left.\left.-\tan\left[\frac{1}{2}(a+b x)\right]^2\right] \sec\left[\frac{1}{2}(a+b x)\right]^2 \tan\left[\frac{1}{2}(a+b x)\right] - \right.\right. \\
& \left.\left.\left.\frac{1}{3+m} (1+m) (1+2m) \operatorname{AppellF1}\left[1+\frac{1+m}{2}, -m, 2+2m, 1+\frac{3+m}{2},\right.\right.\right.\right. \\
& \left.\left.\left.\left.-\tan\left[\frac{1}{2}(a+b x)\right]^2\right]\right)\right)
\end{aligned}$$

$$\begin{aligned}
& \operatorname{Tan}\left[\frac{1}{2} (a + b x)\right]^2, -\operatorname{Tan}\left[\frac{1}{2} (a + b x)\right]^2] \\
& \operatorname{Sec}\left[\frac{1}{2} (a + b x)\right]^2 \operatorname{Tan}\left[\frac{1}{2} (a + b x)\right] \left(1 + \operatorname{Tan}\left[\frac{1}{2} (a + b x)\right]^2\right)^3\Big) / \\
& \left((3 + m) \operatorname{AppellF1}\left[\frac{1+m}{2}, -m, 1+2m, \frac{3+m}{2}, \operatorname{Tan}\left[\frac{1}{2} (a + b x)\right]^2, -\operatorname{Tan}\left[\frac{1}{2} (a + b x)\right]^2\right] - \right. \\
& 2 \left(m \operatorname{AppellF1}\left[\frac{3+m}{2}, 1-m, 1+2m, \frac{5+m}{2}, \operatorname{Tan}\left[\frac{1}{2} (a + b x)\right]^2, -\operatorname{Tan}\left[\frac{1}{2} (a + b x)\right]^2\right] + \right. \\
& (1+2m) \operatorname{AppellF1}\left[\frac{3+m}{2}, -m, 2(1+m), \frac{5+m}{2}, \right. \\
& \left.\left.\operatorname{Tan}\left[\frac{1}{2} (a + b x)\right]^2, -\operatorname{Tan}\left[\frac{1}{2} (a + b x)\right]^2\right]\right) \operatorname{Tan}\left[\frac{1}{2} (a + b x)\right]^2\Big) + \\
& \left(12 \operatorname{AppellF1}\left[\frac{1+m}{2}, -m, 3+2m, \frac{3+m}{2}, \operatorname{Tan}\left[\frac{1}{2} (a + b x)\right]^2, -\operatorname{Tan}\left[\frac{1}{2} (a + b x)\right]^2\right]\right. \\
& \left.\operatorname{Sec}\left[\frac{1}{2} (a + b x)\right]^2 \operatorname{Tan}\left[\frac{1}{2} (a + b x)\right]\right) / \\
& \left((3 + m) \operatorname{AppellF1}\left[\frac{1+m}{2}, -m, 3+2m, \frac{3+m}{2}, \operatorname{Tan}\left[\frac{1}{2} (a + b x)\right]^2, -\operatorname{Tan}\left[\frac{1}{2} (a + b x)\right]^2\right] - \right. \\
& 2 \left(m \operatorname{AppellF1}\left[\frac{3+m}{2}, 1-m, 3+2m, \frac{5+m}{2}, \operatorname{Tan}\left[\frac{1}{2} (a + b x)\right]^2, -\operatorname{Tan}\left[\frac{1}{2} (a + b x)\right]^2\right] + \right. \\
& (3+2m) \operatorname{AppellF1}\left[\frac{3+m}{2}, -m, 2(2+m), \frac{5+m}{2}, \right. \\
& \left.\left.\operatorname{Tan}\left[\frac{1}{2} (a + b x)\right]^2, -\operatorname{Tan}\left[\frac{1}{2} (a + b x)\right]^2\right]\right) \operatorname{Tan}\left[\frac{1}{2} (a + b x)\right]^2\Big) + \\
& \left(12 \left(-\frac{1}{3+m}m(1+m) \operatorname{AppellF1}\left[1+\frac{1+m}{2}, 1-m, 3+2m, 1+\frac{3+m}{2}, \right.\right.\right. \\
& \left.\left.\left.\operatorname{Tan}\left[\frac{1}{2} (a + b x)\right]^2, -\operatorname{Tan}\left[\frac{1}{2} (a + b x)\right]^2\right]\right) \operatorname{Sec}\left[\frac{1}{2} (a + b x)\right]^2 \operatorname{Tan}\left[\frac{1}{2} (a + b x)\right] - \right. \\
& \frac{1}{3+m}(1+m)(3+2m) \operatorname{AppellF1}\left[1+\frac{1+m}{2}, -m, 4+2m, 1+\frac{3+m}{2}, \right. \\
& \left.\left.\operatorname{Tan}\left[\frac{1}{2} (a + b x)\right]^2, -\operatorname{Tan}\left[\frac{1}{2} (a + b x)\right]^2\right]\right) \\
& \operatorname{Sec}\left[\frac{1}{2} (a + b x)\right]^2 \operatorname{Tan}\left[\frac{1}{2} (a + b x)\right] \left(1 + \operatorname{Tan}\left[\frac{1}{2} (a + b x)\right]^2\right)\Big) / \\
& \left((3 + m) \operatorname{AppellF1}\left[\frac{1+m}{2}, -m, 3+2m, \frac{3+m}{2}, \operatorname{Tan}\left[\frac{1}{2} (a + b x)\right]^2, -\operatorname{Tan}\left[\frac{1}{2} (a + b x)\right]^2\right] - \right. \\
& 2 \left(m \operatorname{AppellF1}\left[\frac{3+m}{2}, 1-m, 3+2m, \frac{5+m}{2}, \operatorname{Tan}\left[\frac{1}{2} (a + b x)\right]^2, -\operatorname{Tan}\left[\frac{1}{2} (a + b x)\right]^2\right] + \right. \\
& (3+2m) \operatorname{AppellF1}\left[\frac{3+m}{2}, -m, 2(2+m), \frac{5+m}{2}, \right. \\
& \left.\left.\operatorname{Tan}\left[\frac{1}{2} (a + b x)\right]^2, -\operatorname{Tan}\left[\frac{1}{2} (a + b x)\right]^2\right]\right) \operatorname{Tan}\left[\frac{1}{2} (a + b x)\right]^2\Big) - \\
& \left(12 \operatorname{AppellF1}\left[\frac{1+m}{2}, -m, 2(1+m), \frac{3+m}{2}, \operatorname{Tan}\left[\frac{1}{2} (a + b x)\right]^2, -\operatorname{Tan}\left[\frac{1}{2} (a + b x)\right]^2\right]\right. \\
& \left.\operatorname{Sec}\left[\frac{1}{2} (a + b x)\right]^2 \operatorname{Tan}\left[\frac{1}{2} (a + b x)\right] \left(1 + \operatorname{Tan}\left[\frac{1}{2} (a + b x)\right]^2\right)\right)\Big)
\end{aligned}$$

$$\begin{aligned}
& \left((3+m) \operatorname{AppellF1} \left[\frac{1+m}{2}, -m, 2(1+m), \frac{3+m}{2}, \tan \left[\frac{1}{2}(a+b x) \right]^2, \right. \right. \\
& \quad \left. \left. -\tan \left[\frac{1}{2}(a+b x) \right]^2 \right] - 2 \left(m \operatorname{AppellF1} \left[\frac{3+m}{2}, 1-m, 2(1+m), \frac{5+m}{2}, \right. \right. \\
& \quad \left. \left. \tan \left[\frac{1}{2}(a+b x) \right]^2, -\tan \left[\frac{1}{2}(a+b x) \right]^2 \right] + 2(1+m) \operatorname{AppellF1} \left[\frac{3+m}{2}, -m, \right. \right. \\
& \quad \left. \left. 3+2m, \frac{5+m}{2}, \tan \left[\frac{1}{2}(a+b x) \right]^2, -\tan \left[\frac{1}{2}(a+b x) \right]^2 \right] \right) \tan \left[\frac{1}{2}(a+b x) \right]^2 \right) - \\
& \left(6 \left(-\frac{1}{3+m}m(1+m) \operatorname{AppellF1} \left[1+\frac{1+m}{2}, 1-m, 2(1+m), 1+\frac{3+m}{2}, \right. \right. \right. \\
& \quad \left. \left. \tan \left[\frac{1}{2}(a+b x) \right]^2, -\tan \left[\frac{1}{2}(a+b x) \right]^2 \right] \sec \left[\frac{1}{2}(a+b x) \right]^2 \tan \left[\frac{1}{2}(a+b x) \right] - \right. \\
& \quad \left. \left. \frac{1}{3+m}2(1+m)^2 \operatorname{AppellF1} \left[1+\frac{1+m}{2}, -m, 1+2(1+m), 1+\frac{3+m}{2}, \right. \right. \right. \\
& \quad \left. \left. \tan \left[\frac{1}{2}(a+b x) \right]^2, -\tan \left[\frac{1}{2}(a+b x) \right]^2 \right] \right. \\
& \quad \left. \sec \left[\frac{1}{2}(a+b x) \right]^2 \tan \left[\frac{1}{2}(a+b x) \right] \right) \left(1+\tan \left[\frac{1}{2}(a+b x) \right]^2 \right)^2 \right) / \\
& \left((3+m) \operatorname{AppellF1} \left[\frac{1+m}{2}, -m, 2(1+m), \frac{3+m}{2}, \tan \left[\frac{1}{2}(a+b x) \right]^2, \right. \right. \\
& \quad \left. \left. -\tan \left[\frac{1}{2}(a+b x) \right]^2 \right] - 2 \left(m \operatorname{AppellF1} \left[\frac{3+m}{2}, 1-m, 2(1+m), \frac{5+m}{2}, \right. \right. \\
& \quad \left. \left. \tan \left[\frac{1}{2}(a+b x) \right]^2, -\tan \left[\frac{1}{2}(a+b x) \right]^2 \right] + 2(1+m) \operatorname{AppellF1} \left[\frac{3+m}{2}, -m, \right. \right. \\
& \quad \left. \left. 3+2m, \frac{5+m}{2}, \tan \left[\frac{1}{2}(a+b x) \right]^2, -\tan \left[\frac{1}{2}(a+b x) \right]^2 \right] \right) \tan \left[\frac{1}{2}(a+b x) \right]^2 \right) - \\
& \left(8 \left(-\frac{1}{3+m}m(1+m) \operatorname{AppellF1} \left[1+\frac{1+m}{2}, 1-m, 2(2+m), 1+\frac{3+m}{2}, \tan \left[\frac{1}{2}(a+b x) \right]^2, \right. \right. \right. \\
& \quad \left. \left. -\tan \left[\frac{1}{2}(a+b x) \right]^2 \right] \sec \left[\frac{1}{2}(a+b x) \right]^2 \tan \left[\frac{1}{2}(a+b x) \right] - \frac{1}{3+m}2(1+m) \right. \\
& \quad \left. (2+m) \operatorname{AppellF1} \left[1+\frac{1+m}{2}, -m, 1+2(2+m), 1+\frac{3+m}{2}, \tan \left[\frac{1}{2}(a+b x) \right]^2, \right. \right. \\
& \quad \left. \left. -\tan \left[\frac{1}{2}(a+b x) \right]^2 \right] \sec \left[\frac{1}{2}(a+b x) \right]^2 \tan \left[\frac{1}{2}(a+b x) \right] \right) \right) / \\
& \left((3+m) \operatorname{AppellF1} \left[\frac{1+m}{2}, -m, 2(2+m), \frac{3+m}{2}, \tan \left[\frac{1}{2}(a+b x) \right]^2, \right. \right. \\
& \quad \left. \left. -\tan \left[\frac{1}{2}(a+b x) \right]^2 \right] - 2 \left(m \operatorname{AppellF1} \left[\frac{3+m}{2}, 1-m, 2(2+m), \frac{5+m}{2}, \right. \right. \\
& \quad \left. \left. \tan \left[\frac{1}{2}(a+b x) \right]^2, -\tan \left[\frac{1}{2}(a+b x) \right]^2 \right] + 2(2+m) \operatorname{AppellF1} \left[\frac{3+m}{2}, -m, \right. \right. \\
& \quad \left. \left. 5+2m, \frac{5+m}{2}, \tan \left[\frac{1}{2}(a+b x) \right]^2, -\tan \left[\frac{1}{2}(a+b x) \right]^2 \right] \right) \tan \left[\frac{1}{2}(a+b x) \right]^2 \right) + \\
& \left(6 \operatorname{AppellF1} \left[\frac{1+m}{2}, -m, 2(1+m), \frac{3+m}{2}, \tan \left[\frac{1}{2}(a+b x) \right]^2, -\tan \left[\frac{1}{2}(a+b x) \right]^2 \right] \right. \\
& \quad \left. \left(1+\tan \left[\frac{1}{2}(a+b x) \right]^2 \right)^2 \left(-2 \left(m \operatorname{AppellF1} \left[\frac{3+m}{2}, 1-m, 2(1+m), \frac{5+m}{2}, \right. \right. \right. \right. \right. \\
& \quad \left. \left. \left. \tan \left[\frac{1}{2}(a+b x) \right]^2, -\tan \left[\frac{1}{2}(a+b x) \right]^2 \right] \right) \right)
\end{aligned}$$

$$\begin{aligned}
& \frac{5+m}{2}, \tan\left[\frac{1}{2}(a+b x)\right]^2, -\tan\left[\frac{1}{2}(a+b x)\right]^2\right) \sec\left[\frac{1}{2}(a+b x)\right]^2 \\
& \tan\left[\frac{1}{2}(a+b x)\right] + (3+m) \left(-\frac{1}{3+m} m (1+m) \text{AppellF1}\left[1+\frac{1+m}{2}, 1-m, 2(2+m),\right.\right. \\
& \left.\left.1+\frac{3+m}{2}, \tan\left[\frac{1}{2}(a+b x)\right]^2, -\tan\left[\frac{1}{2}(a+b x)\right]^2\right] \sec\left[\frac{1}{2}(a+b x)\right]^2\right. \\
& \tan\left[\frac{1}{2}(a+b x)\right] - \frac{1}{3+m} 2(1+m)(2+m) \text{AppellF1}\left[1+\frac{1+m}{2}, -m,\right. \\
& \left.1+2(2+m), 1+\frac{3+m}{2}, \tan\left[\frac{1}{2}(a+b x)\right]^2, -\tan\left[\frac{1}{2}(a+b x)\right]^2\right] \\
& \sec\left[\frac{1}{2}(a+b x)\right]^2 \tan\left[\frac{1}{2}(a+b x)\right]\Big) - 2 \tan\left[\frac{1}{2}(a+b x)\right]^2 \\
& \left(m \left(-\frac{1}{5+m} 2(2+m)(3+m) \text{AppellF1}\left[1+\frac{3+m}{2}, 1-m, 1+2(2+m),\right.\right. \right. \\
& \left.\left.\left.1+\frac{5+m}{2}, \tan\left[\frac{1}{2}(a+b x)\right]^2, -\tan\left[\frac{1}{2}(a+b x)\right]^2\right] \sec\left[\frac{1}{2}(a+b x)\right]^2\right. \right. \\
& \left.\left.\tan\left[\frac{1}{2}(a+b x)\right] + \frac{1}{5+m}(1-m)(3+m) \text{AppellF1}\left[1+\frac{3+m}{2},\right.\right. \\
& \left.\left.2-m, 2(2+m), 1+\frac{5+m}{2}, \tan\left[\frac{1}{2}(a+b x)\right]^2, -\tan\left[\frac{1}{2}(a+b x)\right]^2\right] \right. \\
& \sec\left[\frac{1}{2}(a+b x)\right]^2 \tan\left[\frac{1}{2}(a+b x)\right]\Big) + 2(2+m) \left(-\frac{1}{5+m}\right. \\
& \left.m(3+m) \text{AppellF1}\left[1+\frac{3+m}{2}, 1-m, 5+2m, 1+\frac{5+m}{2}, \tan\left[\frac{1}{2}(a+b x)\right]^2,\right.\right. \\
& \left.\left.-\tan\left[\frac{1}{2}(a+b x)\right]^2\right] \sec\left[\frac{1}{2}(a+b x)\right]^2 \tan\left[\frac{1}{2}(a+b x)\right] - \frac{1}{5+m}(3+m)\right. \\
& \left.(5+2m) \text{AppellF1}\left[1+\frac{3+m}{2}, -m, 6+2m, 1+\frac{5+m}{2}, \tan\left[\frac{1}{2}(a+b x)\right]^2,\right.\right. \\
& \left.\left.-\tan\left[\frac{1}{2}(a+b x)\right]^2\right] \sec\left[\frac{1}{2}(a+b x)\right]^2 \tan\left[\frac{1}{2}(a+b x)\right]\right)\Big)\Big)\Big) \\
& \left((3+m) \text{AppellF1}\left[\frac{1+m}{2}, -m, 2(2+m), \frac{3+m}{2}, \tan\left[\frac{1}{2}(a+b x)\right]^2,\right.\right. \\
& \left.\left.-\tan\left[\frac{1}{2}(a+b x)\right]^2\right] - 2 \left(m \text{AppellF1}\left[\frac{3+m}{2}, 1-m, 2(2+m), \frac{5+m}{2},\right.\right. \right. \\
& \left.\left.\left.\tan\left[\frac{1}{2}(a+b x)\right]^2, -\tan\left[\frac{1}{2}(a+b x)\right]^2\right] + 2(2+m) \text{AppellF1}\left[\frac{3+m}{2}, -m, 5+\right.\right. \\
& \left.\left.2m, \frac{5+m}{2}, \tan\left[\frac{1}{2}(a+b x)\right]^2, -\tan\left[\frac{1}{2}(a+b x)\right]^2\right]\right) \tan\left[\frac{1}{2}(a+b x)\right]^2\right)^2 - \\
& \left(\text{AppellF1}\left[\frac{1+m}{2}, -m, 1+2m, \frac{3+m}{2}, \tan\left[\frac{1}{2}(a+b x)\right]^2, -\tan\left[\frac{1}{2}(a+b x)\right]^2\right]\right. \\
& \left.\left(1+\tan\left[\frac{1}{2}(a+b x)\right]^2\right)^3 \left(-2 \left(m \text{AppellF1}\left[\frac{3+m}{2}, 1-m, 1+2m, \frac{5+m}{2},\right.\right. \right. \right. \\
& \left.\left.\left.\tan\left[\frac{1}{2}(a+b x)\right]^2, -\tan\left[\frac{1}{2}(a+b x)\right]^2\right] + (1+2m) \text{AppellF1}\left[\frac{3+m}{2},\right.\right. \right. \\
& \left.\left.-m, 2(1+m), \frac{5+m}{2}, \tan\left[\frac{1}{2}(a+b x)\right]^2, -\tan\left[\frac{1}{2}(a+b x)\right]^2\right]\right)\right)
\end{aligned}$$

$$\begin{aligned}
& \operatorname{Sec}\left[\frac{1}{2}(a+b x)\right]^2 \tan\left[\frac{1}{2}(a+b x)\right] + (3+m) \left(-\frac{1}{3+m} m (1+m) \operatorname{AppellF1}\left[1+\frac{1+m}{2}, 1-m, 1+2m, 1+\frac{3+m}{2}, \tan\left[\frac{1}{2}(a+b x)\right]^2, -\tan\left[\frac{1}{2}(a+b x)\right]^2\right]\right. \\
& \operatorname{Sec}\left[\frac{1}{2}(a+b x)\right]^2 \tan\left[\frac{1}{2}(a+b x)\right] - \frac{1}{3+m} (1+m) (1+2m) \operatorname{AppellF1}\left[1+\frac{1+m}{2}, -m, 2+2m, 1+\frac{3+m}{2}, \tan\left[\frac{1}{2}(a+b x)\right]^2, -\tan\left[\frac{1}{2}(a+b x)\right]^2\right] \\
& \left. \operatorname{Sec}\left[\frac{1}{2}(a+b x)\right]^2 \tan\left[\frac{1}{2}(a+b x)\right]\right) - 2 \tan\left[\frac{1}{2}(a+b x)\right]^2 \\
& \left(m \left(-\frac{1}{5+m} (3+m) (1+2m) \operatorname{AppellF1}\left[1+\frac{3+m}{2}, 1-m, 2+2m, 1+\frac{5+m}{2}, \tan\left[\frac{1}{2}(a+b x)\right]^2, -\tan\left[\frac{1}{2}(a+b x)\right]^2\right] \operatorname{Sec}\left[\frac{1}{2}(a+b x)\right]^2 \tan\left[\frac{1}{2}(a+b x)\right] + \frac{1}{5+m} (1-m) (3+m) \operatorname{AppellF1}\left[1+\frac{3+m}{2}, 2-m, 1+2m, 1+\frac{5+m}{2}, \tan\left[\frac{1}{2}(a+b x)\right]^2, -\tan\left[\frac{1}{2}(a+b x)\right]^2\right] \operatorname{Sec}\left[\frac{1}{2}(a+b x)\right]^2 \tan\left[\frac{1}{2}(a+b x)\right]\right) + (1+2m) \left(-\frac{1}{5+m} m (3+m) \operatorname{AppellF1}\left[1+\frac{3+m}{2}, 1-m, 2(1+m), 1+\frac{5+m}{2}, \tan\left[\frac{1}{2}(a+b x)\right]^2, -\tan\left[\frac{1}{2}(a+b x)\right]^2\right] \operatorname{Sec}\left[\frac{1}{2}(a+b x)\right]^2 \tan\left[\frac{1}{2}(a+b x)\right] - \frac{1}{5+m} 2 (1+m) (3+m) \operatorname{AppellF1}\left[1+\frac{3+m}{2}, -m, 1+2(1+m), 1+\frac{5+m}{2}, \tan\left[\frac{1}{2}(a+b x)\right]^2, -\tan\left[\frac{1}{2}(a+b x)\right]^2\right] \operatorname{Sec}\left[\frac{1}{2}(a+b x)\right]^2 \tan\left[\frac{1}{2}(a+b x)\right]\right)\right) \right) / \\
& \left((3+m) \operatorname{AppellF1}\left[\frac{1+m}{2}, -m, 1+2m, \frac{3+m}{2}, \tan\left[\frac{1}{2}(a+b x)\right]^2, -\tan\left[\frac{1}{2}(a+b x)\right]^2\right] - 2 \left(m \operatorname{AppellF1}\left[\frac{3+m}{2}, 1-m, 1+2m, \frac{5+m}{2}, \tan\left[\frac{1}{2}(a+b x)\right]^2, -\tan\left[\frac{1}{2}(a+b x)\right]^2\right] + (1+2m) \operatorname{AppellF1}\left[\frac{3+m}{2}, -m, 2(1+m), \frac{5+m}{2}, \tan\left[\frac{1}{2}(a+b x)\right]^2, -\tan\left[\frac{1}{2}(a+b x)\right]^2\right] \right) \tan\left[\frac{1}{2}(a+b x)\right]^2 \right) - \\
& \left(12 \operatorname{AppellF1}\left[\frac{1+m}{2}, -m, 3+2m, \frac{3+m}{2}, \tan\left[\frac{1}{2}(a+b x)\right]^2, -\tan\left[\frac{1}{2}(a+b x)\right]^2\right] \left(1 + \tan\left[\frac{1}{2}(a+b x)\right]^2 \right) \left(-2 \left(m \operatorname{AppellF1}\left[\frac{3+m}{2}, 1-m, 3+2m, \frac{5+m}{2}, \tan\left[\frac{1}{2}(a+b x)\right]^2, -\tan\left[\frac{1}{2}(a+b x)\right]^2\right] + (3+2m) \operatorname{AppellF1}\left[\frac{3+m}{2}, -m, 2(2+m), \frac{5+m}{2}, \tan\left[\frac{1}{2}(a+b x)\right]^2, -\tan\left[\frac{1}{2}(a+b x)\right]^2\right] \right) \right) \right. \\
& \left. \operatorname{Sec}\left[\frac{1}{2}(a+b x)\right]^2 \tan\left[\frac{1}{2}(a+b x)\right] + (3+m) \left(-\frac{1}{3+m} m (1+m) \operatorname{AppellF1}\left[1+\frac{1+m}{2}, 1-m, 3+2m, 1+\frac{3+m}{2}, \tan\left[\frac{1}{2}(a+b x)\right]^2, -\tan\left[\frac{1}{2}(a+b x)\right]^2\right]\right)\right)
\end{aligned}$$

$$\begin{aligned}
& 1 + \frac{1+m}{2}, 1-m, 3+2m, 1 + \frac{3+m}{2}, \tan\left[\frac{1}{2}(a+b x)\right]^2, -\tan\left[\frac{1}{2}(a+b x)\right]^2 \\
& \sec\left[\frac{1}{2}(a+b x)\right]^2 \tan\left[\frac{1}{2}(a+b x)\right] - \frac{1}{3+m} (1+m) (3+2m) \text{AppellF1}\left[1 + \frac{1+m}{2}, -m, 4+2m, 1 + \frac{3+m}{2}, \tan\left[\frac{1}{2}(a+b x)\right]^2, -\tan\left[\frac{1}{2}(a+b x)\right]^2\right] \\
& \sec\left[\frac{1}{2}(a+b x)\right]^2 \tan\left[\frac{1}{2}(a+b x)\right] \Big) - 2 \tan\left[\frac{1}{2}(a+b x)\right]^2 \\
& \left(m \left(-\frac{1}{5+m} (3+m) (3+2m) \text{AppellF1}\left[1 + \frac{3+m}{2}, 1-m, 4+2m, 1 + \frac{5+m}{2}, \tan\left[\frac{1}{2}(a+b x)\right]^2, -\tan\left[\frac{1}{2}(a+b x)\right]^2\right] \sec\left[\frac{1}{2}(a+b x)\right]^2 \tan\left[\frac{1}{2}(a+b x)\right] + \frac{1}{5+m} (1-m) (3+m) \text{AppellF1}\left[1 + \frac{3+m}{2}, 2-m, 3+2m, 1 + \frac{5+m}{2}, \tan\left[\frac{1}{2}(a+b x)\right]^2, -\tan\left[\frac{1}{2}(a+b x)\right]^2\right] \sec\left[\frac{1}{2}(a+b x)\right]^2 \tan\left[\frac{1}{2}(a+b x)\right] \right) + (3+2m) \left(-\frac{1}{5+m} m (3+m) \text{AppellF1}\left[1 + \frac{3+m}{2}, 1-m, 2+m, 1 + \frac{5+m}{2}, \tan\left[\frac{1}{2}(a+b x)\right]^2, -\tan\left[\frac{1}{2}(a+b x)\right]^2\right] \sec\left[\frac{1}{2}(a+b x)\right]^2 \tan\left[\frac{1}{2}(a+b x)\right] - \frac{1}{5+m} 2 (2+m) (3+m) \text{AppellF1}\left[1 + \frac{3+m}{2}, -m, 1+2(2+m), 1 + \frac{5+m}{2}, \tan\left[\frac{1}{2}(a+b x)\right]^2, -\tan\left[\frac{1}{2}(a+b x)\right]^2\right] \sec\left[\frac{1}{2}(a+b x)\right]^2 \tan\left[\frac{1}{2}(a+b x)\right] \right) \right) \Big) \Big) \Big) / \\
& \left((3+m) \text{AppellF1}\left[\frac{1+m}{2}, -m, 3+2m, \frac{3+m}{2}, \tan\left[\frac{1}{2}(a+b x)\right]^2, -\tan\left[\frac{1}{2}(a+b x)\right]^2\right] - 2 \left(m \text{AppellF1}\left[\frac{3+m}{2}, 1-m, 3+2m, \frac{5+m}{2}, \tan\left[\frac{1}{2}(a+b x)\right]^2, -\tan\left[\frac{1}{2}(a+b x)\right]^2\right] + (3+2m) \text{AppellF1}\left[\frac{3+m}{2}, -m, 2(2+m), \frac{5+m}{2}, \tan\left[\frac{1}{2}(a+b x)\right]^2, -\tan\left[\frac{1}{2}(a+b x)\right]^2\right] \tan\left[\frac{1}{2}(a+b x)\right]^2 \right) \right) \Big) \Big)
\end{aligned}$$

Problem 188: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \cos[a+b x]^2 \sin[2 a+2 b x]^m dx$$

Optimal (type 5, 85 leaves, 2 steps):

$$\begin{aligned}
& -\frac{1}{b(3+m)} \cos[a+b x]^2 \cot[a+b x] \\
& \text{Hypergeometric2F1}\left[\frac{1-m}{2}, \frac{3+m}{2}, \frac{5+m}{2}, \cos[a+b x]^2\right] (\sin[a+b x]^2)^{\frac{1-m}{2}} \sin[2 a+2 b x]^m
\end{aligned}$$

Result (type 6, 7926 leaves) :

$$\begin{aligned}
& \left(2^{1+2m} (3+m) \cos[a+b x]^2 \sin[2(a+b x)]^m \right. \\
& \left. \tan\left[\frac{1}{2}(a+b x)\right] \left(\frac{\tan\left[\frac{1}{2}(a+b x)\right] - \tan\left[\frac{1}{2}(a+b x)\right]^3}{(1+\tan\left[\frac{1}{2}(a+b x)\right]^2)^2} \right)^m \right. \\
& \left. \left(\left(\text{AppellF1}\left[\frac{1+m}{2}, -m, 1+2m, \frac{3+m}{2}, \tan\left[\frac{1}{2}(a+b x)\right]^2, -\tan\left[\frac{1}{2}(a+b x)\right]^2 \right] \right. \right. \right. \\
& \left. \left. \left. \left(1+\tan\left[\frac{1}{2}(a+b x)\right]^2 \right)^2 \right) \right. \\
& \left. \left((3+m) \text{AppellF1}\left[\frac{1+m}{2}, -m, 1+2m, \frac{3+m}{2}, \tan\left[\frac{1}{2}(a+b x)\right]^2, -\tan\left[\frac{1}{2}(a+b x)\right]^2 \right] - \right. \right. \\
& \left. \left. 2 \left(m \text{AppellF1}\left[\frac{3+m}{2}, 1-m, 1+2m, \frac{5+m}{2}, \tan\left[\frac{1}{2}(a+b x)\right]^2, -\tan\left[\frac{1}{2}(a+b x)\right]^2 \right] + \right. \right. \\
& \left. \left. (1+2m) \text{AppellF1}\left[\frac{3+m}{2}, -m, 2(1+m), \frac{5+m}{2}, \right. \right. \right. \\
& \left. \left. \left. \tan\left[\frac{1}{2}(a+b x)\right]^2, -\tan\left[\frac{1}{2}(a+b x)\right]^2 \right] \right) \tan\left[\frac{1}{2}(a+b x)\right]^2 \right) + \\
& \left. \left(4 \text{AppellF1}\left[\frac{1+m}{2}, -m, 3+2m, \frac{3+m}{2}, \tan\left[\frac{1}{2}(a+b x)\right]^2, -\tan\left[\frac{1}{2}(a+b x)\right]^2 \right] \right) \right. \\
& \left. \left((3+m) \text{AppellF1}\left[\frac{1+m}{2}, -m, 3+2m, \frac{3+m}{2}, \tan\left[\frac{1}{2}(a+b x)\right]^2, -\tan\left[\frac{1}{2}(a+b x)\right]^2 \right] - \right. \right. \\
& \left. \left. 2 \left(m \text{AppellF1}\left[\frac{3+m}{2}, 1-m, 3+2m, \frac{5+m}{2}, \tan\left[\frac{1}{2}(a+b x)\right]^2, -\tan\left[\frac{1}{2}(a+b x)\right]^2 \right] + \right. \right. \\
& \left. \left. (3+2m) \text{AppellF1}\left[\frac{3+m}{2}, -m, 2(2+m), \frac{5+m}{2}, \right. \right. \right. \\
& \left. \left. \left. \tan\left[\frac{1}{2}(a+b x)\right]^2, -\tan\left[\frac{1}{2}(a+b x)\right]^2 \right] \right) \tan\left[\frac{1}{2}(a+b x)\right]^2 \right) - \\
& \left. \left(4 \text{AppellF1}\left[\frac{1+m}{2}, -m, 2(1+m), \frac{3+m}{2}, \tan\left[\frac{1}{2}(a+b x)\right]^2, -\tan\left[\frac{1}{2}(a+b x)\right]^2 \right] \right. \right. \\
& \left. \left. \left(1+\tan\left[\frac{1}{2}(a+b x)\right]^2 \right) \right) \right. \\
& \left. \left((3+m) \text{AppellF1}\left[\frac{1+m}{2}, -m, 2(1+m), \frac{3+m}{2}, \tan\left[\frac{1}{2}(a+b x)\right]^2, -\tan\left[\frac{1}{2}(a+b x)\right]^2 \right] - \right. \right. \\
& \left. \left. 2 \left(m \text{AppellF1}\left[\frac{3+m}{2}, 1-m, 2(1+m), \frac{5+m}{2}, \tan\left[\frac{1}{2}(a+b x)\right]^2, -\tan\left[\frac{1}{2}(a+b x)\right]^2 \right] + \right. \right. \\
& \left. \left. 2(1+m) \text{AppellF1}\left[\frac{3+m}{2}, -m, 3+2m, \frac{5+m}{2}, \tan\left[\frac{1}{2}(a+b x)\right]^2, -\tan\left[\frac{1}{2}(a+b x)\right]^2 \right] \right) \right)
\end{aligned}$$

$$\begin{aligned}
& \left. \left(\frac{\tan\left[\frac{1}{2}(\mathbf{a} + \mathbf{b}x)\right]^2\right) \right) \Bigg/ \left(\mathbf{b} (1+\mathbf{m}) \left(1 + \tan\left[\frac{1}{2}(\mathbf{a} + \mathbf{b}x)\right]^2\right)^3 \right. \\
& \left. - \frac{1}{(1+\mathbf{m}) \left(1 + \tan\left[\frac{1}{2}(\mathbf{a} + \mathbf{b}x)\right]^2\right)^4} 3 \times 2^{1+2\mathbf{m}} (3+\mathbf{m}) \sec\left[\frac{1}{2}(\mathbf{a} + \mathbf{b}x)\right]^2 \right. \\
& \left. \tan\left[\frac{1}{2}(\mathbf{a} + \mathbf{b}x)\right]^2 \left(\frac{\tan\left[\frac{1}{2}(\mathbf{a} + \mathbf{b}x)\right] - \tan\left[\frac{1}{2}(\mathbf{a} + \mathbf{b}x)\right]^3}{\left(1 + \tan\left[\frac{1}{2}(\mathbf{a} + \mathbf{b}x)\right]^2\right)^2} \right)^{\mathbf{m}} \right. \\
& \left. \left(\left(\text{AppellF1}\left[\frac{1+\mathbf{m}}{2}, -\mathbf{m}, 1+2\mathbf{m}, \frac{3+\mathbf{m}}{2}, \tan\left[\frac{1}{2}(\mathbf{a} + \mathbf{b}x)\right]^2, -\tan\left[\frac{1}{2}(\mathbf{a} + \mathbf{b}x)\right]^2 \right] \right. \right. \\
& \left. \left. \left(1 + \tan\left[\frac{1}{2}(\mathbf{a} + \mathbf{b}x)\right]^2 \right)^2 \right) \Bigg/ \left((3+\mathbf{m}) \text{AppellF1}\left[\frac{1+\mathbf{m}}{2}, -\mathbf{m}, 1+2\mathbf{m}, \frac{3+\mathbf{m}}{2}, \right. \right. \\
& \left. \left. \tan\left[\frac{1}{2}(\mathbf{a} + \mathbf{b}x)\right]^2, -\tan\left[\frac{1}{2}(\mathbf{a} + \mathbf{b}x)\right]^2 \right] - 2 \left(\mathbf{m} \text{AppellF1}\left[\frac{3+\mathbf{m}}{2}, 1-\mathbf{m}, 1+2\mathbf{m}, \right. \right. \\
& \left. \left. \frac{5+\mathbf{m}}{2}, \tan\left[\frac{1}{2}(\mathbf{a} + \mathbf{b}x)\right]^2, -\tan\left[\frac{1}{2}(\mathbf{a} + \mathbf{b}x)\right]^2 \right] + (1+2\mathbf{m}) \text{AppellF1}\left[\frac{3+\mathbf{m}}{2}, -\mathbf{m}, \right. \right. \\
& \left. \left. 2(1+\mathbf{m}), \frac{5+\mathbf{m}}{2}, \tan\left[\frac{1}{2}(\mathbf{a} + \mathbf{b}x)\right]^2, -\tan\left[\frac{1}{2}(\mathbf{a} + \mathbf{b}x)\right]^2 \right) \tan\left[\frac{1}{2}(\mathbf{a} + \mathbf{b}x)\right]^2 \right) + \right. \\
& \left. \left(4 \text{AppellF1}\left[\frac{1+\mathbf{m}}{2}, -\mathbf{m}, 3+2\mathbf{m}, \frac{3+\mathbf{m}}{2}, \tan\left[\frac{1}{2}(\mathbf{a} + \mathbf{b}x)\right]^2, -\tan\left[\frac{1}{2}(\mathbf{a} + \mathbf{b}x)\right]^2 \right] \right) \Bigg/ \right. \\
& \left. \left((3+\mathbf{m}) \text{AppellF1}\left[\frac{1+\mathbf{m}}{2}, -\mathbf{m}, 3+2\mathbf{m}, \frac{3+\mathbf{m}}{2}, \tan\left[\frac{1}{2}(\mathbf{a} + \mathbf{b}x)\right]^2, -\tan\left[\frac{1}{2}(\mathbf{a} + \mathbf{b}x)\right]^2 \right] - \right. \right. \\
& \left. \left. 2 \left(\mathbf{m} \text{AppellF1}\left[\frac{3+\mathbf{m}}{2}, 1-\mathbf{m}, 3+2\mathbf{m}, \frac{5+\mathbf{m}}{2}, \tan\left[\frac{1}{2}(\mathbf{a} + \mathbf{b}x)\right]^2, -\tan\left[\frac{1}{2}(\mathbf{a} + \mathbf{b}x)\right]^2 \right] + \right. \right. \\
& \left. \left. (3+2\mathbf{m}) \text{AppellF1}\left[\frac{3+\mathbf{m}}{2}, -\mathbf{m}, 2(2+\mathbf{m}), \frac{5+\mathbf{m}}{2}, \right. \right. \right. \\
& \left. \left. \left. \tan\left[\frac{1}{2}(\mathbf{a} + \mathbf{b}x)\right]^2, -\tan\left[\frac{1}{2}(\mathbf{a} + \mathbf{b}x)\right]^2 \right) \tan\left[\frac{1}{2}(\mathbf{a} + \mathbf{b}x)\right]^2 \right) - \right. \\
& \left. \left(4 \text{AppellF1}\left[\frac{1+\mathbf{m}}{2}, -\mathbf{m}, 2(1+\mathbf{m}), \frac{3+\mathbf{m}}{2}, \tan\left[\frac{1}{2}(\mathbf{a} + \mathbf{b}x)\right]^2, -\tan\left[\frac{1}{2}(\mathbf{a} + \mathbf{b}x)\right]^2 \right] \right. \\
& \left. \left. \left(1 + \tan\left[\frac{1}{2}(\mathbf{a} + \mathbf{b}x)\right]^2 \right) \right) \Bigg/ \left((3+\mathbf{m}) \text{AppellF1}\left[\frac{1+\mathbf{m}}{2}, -\mathbf{m}, 2(1+\mathbf{m}), \frac{3+\mathbf{m}}{2}, \right. \right. \right. \\
& \left. \left. \left. \tan\left[\frac{1}{2}(\mathbf{a} + \mathbf{b}x)\right]^2, -\tan\left[\frac{1}{2}(\mathbf{a} + \mathbf{b}x)\right]^2 \right] - 2 \left(\mathbf{m} \text{AppellF1}\left[\frac{3+\mathbf{m}}{2}, 1-\mathbf{m}, 2(1+\mathbf{m}), \right. \right. \right. \\
& \left. \left. \left. \frac{5+\mathbf{m}}{2}, \tan\left[\frac{1}{2}(\mathbf{a} + \mathbf{b}x)\right]^2, -\tan\left[\frac{1}{2}(\mathbf{a} + \mathbf{b}x)\right]^2 \right] + 2(1+\mathbf{m}) \text{AppellF1}\left[\frac{3+\mathbf{m}}{2}, -\mathbf{m}, \right. \right. \right. \\
& \left. \left. \left. 3+2\mathbf{m}, \frac{5+\mathbf{m}}{2}, \tan\left[\frac{1}{2}(\mathbf{a} + \mathbf{b}x)\right]^2, -\tan\left[\frac{1}{2}(\mathbf{a} + \mathbf{b}x)\right]^2 \right) \tan\left[\frac{1}{2}(\mathbf{a} + \mathbf{b}x)\right]^2 \right) + \right. \\
& \left. \frac{1}{(1+\mathbf{m}) \left(1 + \tan\left[\frac{1}{2}(\mathbf{a} + \mathbf{b}x)\right]^2\right)^3} 2^{2\mathbf{m}} (3+\mathbf{m}) \sec\left[\frac{1}{2}(\mathbf{a} + \mathbf{b}x)\right]^2 \right. \\
& \left. \left(\frac{\tan\left[\frac{1}{2}(\mathbf{a} + \mathbf{b}x)\right] - \tan\left[\frac{1}{2}(\mathbf{a} + \mathbf{b}x)\right]^3}{\left(1 + \tan\left[\frac{1}{2}(\mathbf{a} + \mathbf{b}x)\right]^2\right)^2} \right)^{\mathbf{m}} \right)
\end{aligned}$$

$$\begin{aligned}
& \left(\left(\text{AppellF1} \left[\frac{1+m}{2}, -m, 1+2m, \frac{3+m}{2}, \tan \left[\frac{1}{2} (a+b x) \right]^2, -\tan \left[\frac{1}{2} (a+b x) \right]^2 \right] \right. \right. \\
& \quad \left. \left. \left(1 + \tan \left[\frac{1}{2} (a+b x) \right]^2 \right)^2 \right) / \left((3+m) \text{AppellF1} \left[\frac{1+m}{2}, -m, 1+2m, \frac{3+m}{2}, \right. \right. \\
& \quad \left. \left. \tan \left[\frac{1}{2} (a+b x) \right]^2, -\tan \left[\frac{1}{2} (a+b x) \right]^2 \right] - 2 \left(m \text{AppellF1} \left[\frac{3+m}{2}, 1-m, 1+2m, \right. \right. \\
& \quad \left. \left. \frac{5+m}{2}, \tan \left[\frac{1}{2} (a+b x) \right]^2, -\tan \left[\frac{1}{2} (a+b x) \right]^2 \right] + (1+2m) \text{AppellF1} \left[\frac{3+m}{2}, -m, \right. \right. \\
& \quad \left. \left. 2(1+m), \frac{5+m}{2}, \tan \left[\frac{1}{2} (a+b x) \right]^2, -\tan \left[\frac{1}{2} (a+b x) \right]^2 \right] \right) \tan \left[\frac{1}{2} (a+b x) \right]^2 \right) + \\
& \quad \left(4 \text{AppellF1} \left[\frac{1+m}{2}, -m, 3+2m, \frac{3+m}{2}, \tan \left[\frac{1}{2} (a+b x) \right]^2, -\tan \left[\frac{1}{2} (a+b x) \right]^2 \right] \right) / \\
& \quad \left((3+m) \text{AppellF1} \left[\frac{1+m}{2}, -m, 3+2m, \frac{3+m}{2}, \tan \left[\frac{1}{2} (a+b x) \right]^2, -\tan \left[\frac{1}{2} (a+b x) \right]^2 \right] - \right. \\
& \quad \left. 2 \left(m \text{AppellF1} \left[\frac{3+m}{2}, 1-m, 3+2m, \frac{5+m}{2}, \tan \left[\frac{1}{2} (a+b x) \right]^2, -\tan \left[\frac{1}{2} (a+b x) \right]^2 \right] + \right. \right. \\
& \quad \left. \left. (3+2m) \text{AppellF1} \left[\frac{3+m}{2}, -m, 2(2+m), \frac{5+m}{2}, \right. \right. \\
& \quad \left. \left. \tan \left[\frac{1}{2} (a+b x) \right]^2, -\tan \left[\frac{1}{2} (a+b x) \right]^2 \right] \right) \tan \left[\frac{1}{2} (a+b x) \right]^2 \right) - \\
& \quad \left(4 \text{AppellF1} \left[\frac{1+m}{2}, -m, 2(1+m), \frac{3+m}{2}, \tan \left[\frac{1}{2} (a+b x) \right]^2, -\tan \left[\frac{1}{2} (a+b x) \right]^2 \right] \right. \\
& \quad \left. \left(1 + \tan \left[\frac{1}{2} (a+b x) \right]^2 \right) \right) / \left((3+m) \text{AppellF1} \left[\frac{1+m}{2}, -m, 2(1+m), \frac{3+m}{2}, \right. \right. \\
& \quad \left. \left. \tan \left[\frac{1}{2} (a+b x) \right]^2, -\tan \left[\frac{1}{2} (a+b x) \right]^2 \right] - 2 \left(m \text{AppellF1} \left[\frac{3+m}{2}, 1-m, 2(1+m), \right. \right. \\
& \quad \left. \left. \frac{5+m}{2}, \tan \left[\frac{1}{2} (a+b x) \right]^2, -\tan \left[\frac{1}{2} (a+b x) \right]^2 \right] + 2(1+m) \text{AppellF1} \left[\frac{3+m}{2}, -m, \right. \right. \\
& \quad \left. \left. 3+2m, \frac{5+m}{2}, \tan \left[\frac{1}{2} (a+b x) \right]^2, -\tan \left[\frac{1}{2} (a+b x) \right]^2 \right] \right) \tan \left[\frac{1}{2} (a+b x) \right]^2 \right) + \\
& \quad \frac{1}{(1+m) \left(1 + \tan \left[\frac{1}{2} (a+b x) \right]^2 \right)^3} 2^{1+2m} m (3+m) \tan \left[\frac{1}{2} (a+b x) \right] \\
& \quad \left(\frac{\tan \left[\frac{1}{2} (a+b x) \right] - \tan \left[\frac{1}{2} (a+b x) \right]^3}{\left(1 + \tan \left[\frac{1}{2} (a+b x) \right]^2 \right)^2} \right)^{-1+m} \\
& \quad \left(\left(\text{AppellF1} \left[\frac{1+m}{2}, -m, 1+2m, \frac{3+m}{2}, \tan \left[\frac{1}{2} (a+b x) \right]^2, -\tan \left[\frac{1}{2} (a+b x) \right]^2 \right] \right. \right. \\
& \quad \left. \left. \left(1 + \tan \left[\frac{1}{2} (a+b x) \right]^2 \right)^2 \right) / \right. \\
& \quad \left. \left((3+m) \text{AppellF1} \left[\frac{1+m}{2}, -m, 1+2m, \frac{3+m}{2}, \tan \left[\frac{1}{2} (a+b x) \right]^2, -\tan \left[\frac{1}{2} (a+b x) \right]^2 \right] - \right. \right. \\
& \quad \left. \left. 2 \left(m \text{AppellF1} \left[\frac{3+m}{2}, 1-m, 1+2m, \frac{5+m}{2}, \tan \left[\frac{1}{2} (a+b x) \right]^2, -\tan \left[\frac{1}{2} (a+b x) \right]^2 \right] + \right. \right. \right. \\
& \quad \left. \left. \left. (1+2m) \text{AppellF1} \left[\frac{3+m}{2}, -m, 2(1+m), \frac{5+m}{2}, \right. \right. \right. \\
& \quad \left. \left. \left. \tan \left[\frac{1}{2} (a+b x) \right]^2, -\tan \left[\frac{1}{2} (a+b x) \right]^2 \right] \right) \right)
\end{aligned}$$

$$\begin{aligned}
& \left. \left(\tan\left[\frac{1}{2}(a+b x)\right]^2, -\tan\left[\frac{1}{2}(a+b x)\right]^2 \right) \tan\left[\frac{1}{2}(a+b x)\right]^2 \right) + \\
& \left(4 \text{AppellF1}\left[\frac{1+m}{2}, -m, 3+2m, \frac{3+m}{2}, \tan\left[\frac{1}{2}(a+b x)\right]^2, -\tan\left[\frac{1}{2}(a+b x)\right]^2 \right] \right) / \\
& \left(\left(3+m\right) \text{AppellF1}\left[\frac{1+m}{2}, -m, 3+2m, \frac{3+m}{2}, \tan\left[\frac{1}{2}(a+b x)\right]^2, -\tan\left[\frac{1}{2}(a+b x)\right]^2 \right] - \right. \\
& \left. 2 \left(m \text{AppellF1}\left[\frac{3+m}{2}, 1-m, 3+2m, \frac{5+m}{2}, \tan\left[\frac{1}{2}(a+b x)\right]^2, -\tan\left[\frac{1}{2}(a+b x)\right]^2 \right] + \right. \right. \\
& \left. \left. (3+2m) \text{AppellF1}\left[\frac{3+m}{2}, -m, 2(2+m), \frac{5+m}{2}, \right. \right. \right. \\
& \left. \left. \left. \tan\left[\frac{1}{2}(a+b x)\right]^2, -\tan\left[\frac{1}{2}(a+b x)\right]^2 \right) \tan\left[\frac{1}{2}(a+b x)\right]^2 \right) - \\
& \left(4 \text{AppellF1}\left[\frac{1+m}{2}, -m, 2(1+m), \frac{3+m}{2}, \tan\left[\frac{1}{2}(a+b x)\right]^2, -\tan\left[\frac{1}{2}(a+b x)\right]^2 \right] \right. \\
& \left. \left(1 + \tan\left[\frac{1}{2}(a+b x)\right]^2 \right) \right) / \\
& \left(\left(3+m\right) \text{AppellF1}\left[\frac{1+m}{2}, -m, 2(1+m), \frac{3+m}{2}, \tan\left[\frac{1}{2}(a+b x)\right]^2, -\tan\left[\frac{1}{2}(a+b x)\right]^2 \right] - \right. \\
& \left. 2 \left(m \text{AppellF1}\left[\frac{3+m}{2}, 1-m, 2(1+m), \frac{5+m}{2}, \tan\left[\frac{1}{2}(a+b x)\right]^2, \right. \right. \right. \\
& \left. \left. \left. -\tan\left[\frac{1}{2}(a+b x)\right]^2 \right] + 2(1+m) \text{AppellF1}\left[\frac{3+m}{2}, -m, 3+2m, \frac{5+m}{2}, \right. \right. \right. \\
& \left. \left. \left. \tan\left[\frac{1}{2}(a+b x)\right]^2, -\tan\left[\frac{1}{2}(a+b x)\right]^2 \right) \tan\left[\frac{1}{2}(a+b x)\right]^2 \right) \right) \\
& \left(\frac{\frac{1}{2} \sec\left[\frac{1}{2}(a+b x)\right]^2 - \frac{3}{2} \sec\left[\frac{1}{2}(a+b x)\right]^2 \tan\left[\frac{1}{2}(a+b x)\right]^2}{\left(1 + \tan\left[\frac{1}{2}(a+b x)\right]^2\right)^2} - \right. \\
& \left. \left(2 \sec\left[\frac{1}{2}(a+b x)\right]^2 \tan\left[\frac{1}{2}(a+b x)\right] \left(\tan\left[\frac{1}{2}(a+b x)\right] - \tan\left[\frac{1}{2}(a+b x)\right]^3 \right) \right) \right) / \\
& \left. \left(1 + \tan\left[\frac{1}{2}(a+b x)\right]^2 \right)^3 \right) + \\
& \frac{1}{(1+m) \left(1 + \tan\left[\frac{1}{2}(a+b x)\right]^2\right)^3} 2^{1+2m} (3+m) \tan\left[\frac{1}{2}(a+b x)\right] \\
& \left(\frac{\tan\left[\frac{1}{2}(a+b x)\right] - \tan\left[\frac{1}{2}(a+b x)\right]^3}{\left(1 + \tan\left[\frac{1}{2}(a+b x)\right]^2\right)^2} \right)^m \\
& \left(\left(2 \text{AppellF1}\left[\frac{1+m}{2}, -m, 1+2m, \frac{3+m}{2}, \tan\left[\frac{1}{2}(a+b x)\right]^2, -\tan\left[\frac{1}{2}(a+b x)\right]^2 \right] \right. \right. \\
& \left. \left. \sec\left[\frac{1}{2}(a+b x)\right]^2 \tan\left[\frac{1}{2}(a+b x)\right] \left(1 + \tan\left[\frac{1}{2}(a+b x)\right]^2 \right) \right) \right) / \\
& \left(\left(3+m\right) \text{AppellF1}\left[\frac{1+m}{2}, -m, 1+2m, \frac{3+m}{2}, \tan\left[\frac{1}{2}(a+b x)\right]^2, -\tan\left[\frac{1}{2}(a+b x)\right]^2 \right] - \right. \\
& \left. 2 \left(m \text{AppellF1}\left[\frac{3+m}{2}, 1-m, 1+2m, \frac{5+m}{2}, \tan\left[\frac{1}{2}(a+b x)\right]^2, -\tan\left[\frac{1}{2}(a+b x)\right]^2 \right] + \right. \right.
\end{aligned}$$

$$\begin{aligned}
& (1 + 2m) \operatorname{AppellF1}\left[\frac{3+m}{2}, -m, 2(1+m), \frac{5+m}{2}, \right. \\
& \quad \left. \tan\left[\frac{1}{2}(a+b x)\right]^2, -\tan\left[\frac{1}{2}(a+b x)\right]^2\right] \tan\left[\frac{1}{2}(a+b x)\right]^2 + \\
& \left(\left(-\frac{1}{3+m}m(1+m) \operatorname{AppellF1}\left[1+\frac{1+m}{2}, 1-m, 1+2m, 1+\frac{3+m}{2}, \tan\left[\frac{1}{2}(a+b x)\right]^2, \right. \right. \right. \\
& \quad \left. \left. \left. -\tan\left[\frac{1}{2}(a+b x)\right]^2\right] \sec\left[\frac{1}{2}(a+b x)\right]^2 \tan\left[\frac{1}{2}(a+b x)\right] - \frac{1}{3+m}(1+m)(1+2m) \right. \\
& \quad \left. \operatorname{AppellF1}\left[1+\frac{1+m}{2}, -m, 2+2m, 1+\frac{3+m}{2}, \tan\left[\frac{1}{2}(a+b x)\right]^2, -\tan\left[\frac{1}{2}(a+b x)\right]^2 \right] \right. \\
& \quad \left. \sec\left[\frac{1}{2}(a+b x)\right]^2 \tan\left[\frac{1}{2}(a+b x)\right] \right) \left(1 + \tan\left[\frac{1}{2}(a+b x)\right]^2 \right)^2 \Bigg) / \\
& \left((3+m) \operatorname{AppellF1}\left[\frac{1+m}{2}, -m, 1+2m, \frac{3+m}{2}, \tan\left[\frac{1}{2}(a+b x)\right]^2, -\tan\left[\frac{1}{2}(a+b x)\right]^2 \right] - \right. \\
& \quad \left. 2 \left(m \operatorname{AppellF1}\left[\frac{3+m}{2}, 1-m, 1+2m, \frac{5+m}{2}, \tan\left[\frac{1}{2}(a+b x)\right]^2, -\tan\left[\frac{1}{2}(a+b x)\right]^2 \right] + \right. \right. \\
& \quad \left. \left. (1+2m) \operatorname{AppellF1}\left[\frac{3+m}{2}, -m, 2(1+m), \frac{5+m}{2}, \right. \right. \right. \\
& \quad \left. \left. \tan\left[\frac{1}{2}(a+b x)\right]^2, -\tan\left[\frac{1}{2}(a+b x)\right]^2\right] \tan\left[\frac{1}{2}(a+b x)\right]^2 \right) + \\
& \quad \left(4 \left(-\frac{1}{3+m}m(1+m) \operatorname{AppellF1}\left[1+\frac{1+m}{2}, 1-m, 3+2m, 1+\frac{3+m}{2}, \tan\left[\frac{1}{2}(a+b x)\right]^2, \right. \right. \right. \\
& \quad \left. \left. \left. -\tan\left[\frac{1}{2}(a+b x)\right]^2\right] \sec\left[\frac{1}{2}(a+b x)\right]^2 \tan\left[\frac{1}{2}(a+b x)\right] - \frac{1}{3+m} \right. \\
& \quad \left. \left. (1+m)(3+2m) \operatorname{AppellF1}\left[1+\frac{1+m}{2}, -m, 4+2m, 1+\frac{3+m}{2}, \tan\left[\frac{1}{2}(a+b x)\right]^2, \right. \right. \right. \\
& \quad \left. \left. \left. -\tan\left[\frac{1}{2}(a+b x)\right]^2\right] \sec\left[\frac{1}{2}(a+b x)\right]^2 \tan\left[\frac{1}{2}(a+b x)\right] \right) \right) \Bigg) / \\
& \left((3+m) \operatorname{AppellF1}\left[\frac{1+m}{2}, -m, 3+2m, \frac{3+m}{2}, \tan\left[\frac{1}{2}(a+b x)\right]^2, -\tan\left[\frac{1}{2}(a+b x)\right]^2 \right] - \right. \\
& \quad \left. 2 \left(m \operatorname{AppellF1}\left[\frac{3+m}{2}, 1-m, 3+2m, \frac{5+m}{2}, \tan\left[\frac{1}{2}(a+b x)\right]^2, -\tan\left[\frac{1}{2}(a+b x)\right]^2 \right] + \right. \right. \\
& \quad \left. \left. (3+2m) \operatorname{AppellF1}\left[\frac{3+m}{2}, -m, 2(2+m), \frac{5+m}{2}, \right. \right. \right. \\
& \quad \left. \left. \tan\left[\frac{1}{2}(a+b x)\right]^2, -\tan\left[\frac{1}{2}(a+b x)\right]^2\right] \tan\left[\frac{1}{2}(a+b x)\right]^2 \right) - \\
& \quad \left(4 \operatorname{AppellF1}\left[\frac{1+m}{2}, -m, 2(1+m), \frac{3+m}{2}, \tan\left[\frac{1}{2}(a+b x)\right]^2, -\tan\left[\frac{1}{2}(a+b x)\right]^2 \right] \right. \\
& \quad \left. \sec\left[\frac{1}{2}(a+b x)\right]^2 \tan\left[\frac{1}{2}(a+b x)\right] \right) \Bigg) / \\
& \left((3+m) \operatorname{AppellF1}\left[\frac{1+m}{2}, -m, 2(1+m), \frac{3+m}{2}, \tan\left[\frac{1}{2}(a+b x)\right]^2, -\tan\left[\frac{1}{2}(a+b x)\right]^2 \right] - \right. \\
& \quad \left. 2 \left(m \operatorname{AppellF1}\left[\frac{3+m}{2}, 1-m, 2(1+m), \frac{5+m}{2}, \tan\left[\frac{1}{2}(a+b x)\right]^2, \right. \right. \right. \\
& \quad \left. \left. \left. -\tan\left[\frac{1}{2}(a+b x)\right]^2\right] + 2(1+m) \operatorname{AppellF1}\left[\frac{3+m}{2}, -m, 3+2m, \frac{5+m}{2}, \right. \right. \right. \\
& \quad \left. \left. \left. \tan\left[\frac{1}{2}(a+b x)\right]^2, -\tan\left[\frac{1}{2}(a+b x)\right]^2\right] \right)
\end{aligned}$$

$$\begin{aligned}
& \left. \left(\tan\left[\frac{1}{2}(a+b x)\right]^2, -\tan\left[\frac{1}{2}(a+b x)\right]^2 \right) \tan\left[\frac{1}{2}(a+b x)\right]^2 \right) - \\
& \left(4 \left(-\frac{1}{3+m} m (1+m) \text{AppellF1}\left[1 + \frac{1+m}{2}, 1-m, 2(1+m), 1 + \frac{3+m}{2}, \right. \right. \right. \right. \\
& \quad \left. \left. \left. \left. \tan\left[\frac{1}{2}(a+b x)\right]^2, -\tan\left[\frac{1}{2}(a+b x)\right]^2 \right] \sec\left[\frac{1}{2}(a+b x)\right]^2 \tan\left[\frac{1}{2}(a+b x)\right] - \right. \right. \\
& \quad \left. \left. \left. \left. \frac{1}{3+m} 2(1+m)^2 \text{AppellF1}\left[1 + \frac{1+m}{2}, -m, 1+2(1+m), 1 + \frac{3+m}{2}, \right. \right. \right. \right. \\
& \quad \left. \left. \left. \left. \tan\left[\frac{1}{2}(a+b x)\right]^2, -\tan\left[\frac{1}{2}(a+b x)\right]^2 \right] \sec\left[\frac{1}{2}(a+b x)\right]^2 \tan\left[\frac{1}{2}(a+b x)\right] \right) \right) \\
& \left(1 + \tan\left[\frac{1}{2}(a+b x)\right]^2 \right) \Big/ \left((3+m) \text{AppellF1}\left[\frac{1+m}{2}, -m, 2(1+m), \frac{3+m}{2}, \right. \right. \\
& \quad \left. \left. \tan\left[\frac{1}{2}(a+b x)\right]^2, -\tan\left[\frac{1}{2}(a+b x)\right]^2 \right] \right) \\
& \tan\left[\frac{1}{2}(a+b x)\right]^2, -\tan\left[\frac{1}{2}(a+b x)\right]^2] - 2 \left(m \text{AppellF1}\left[\frac{3+m}{2}, 1-m, 2(1+m), \right. \right. \\
& \quad \left. \left. \frac{5+m}{2}, \tan\left[\frac{1}{2}(a+b x)\right]^2, -\tan\left[\frac{1}{2}(a+b x)\right]^2 \right] + 2(1+m) \text{AppellF1}\left[\frac{3+m}{2}, -m, \right. \right. \\
& \quad \left. \left. 3+2m, \frac{5+m}{2}, \tan\left[\frac{1}{2}(a+b x)\right]^2, -\tan\left[\frac{1}{2}(a+b x)\right]^2 \right] \right) \tan\left[\frac{1}{2}(a+b x)\right]^2 \Big) + \\
& \left(4 \text{AppellF1}\left[\frac{1+m}{2}, -m, 2(1+m), \frac{3+m}{2}, \tan\left[\frac{1}{2}(a+b x)\right]^2, -\tan\left[\frac{1}{2}(a+b x)\right]^2 \right] \right. \\
& \quad \left(1 + \tan\left[\frac{1}{2}(a+b x)\right]^2 \right) \left(-2 \left(m \text{AppellF1}\left[\frac{3+m}{2}, 1-m, 2(1+m), \frac{5+m}{2}, \right. \right. \right. \right. \\
& \quad \left. \left. \left. \left. \tan\left[\frac{1}{2}(a+b x)\right]^2, -\tan\left[\frac{1}{2}(a+b x)\right]^2 \right] + 2(1+m) \text{AppellF1}\left[\frac{3+m}{2}, -m, \right. \right. \right. \\
& \quad \left. \left. \left. 3+2m, \frac{5+m}{2}, \tan\left[\frac{1}{2}(a+b x)\right]^2, -\tan\left[\frac{1}{2}(a+b x)\right]^2 \right] \right) \sec\left[\frac{1}{2}(a+b x)\right]^2 \right. \\
& \quad \left. \tan\left[\frac{1}{2}(a+b x)\right] + (3+m) \left(-\frac{1}{3+m} m (1+m) \text{AppellF1}\left[1 + \frac{1+m}{2}, 1-m, 2(1+m), \right. \right. \right. \\
& \quad \left. \left. \left. 1 + \frac{3+m}{2}, \tan\left[\frac{1}{2}(a+b x)\right]^2, -\tan\left[\frac{1}{2}(a+b x)\right]^2 \right] \sec\left[\frac{1}{2}(a+b x)\right]^2 \tan\left[\frac{1}{2}(a+b x)\right] \right. \right. \\
& \quad \left. \left. - \frac{1}{3+m} 2(1+m)^2 \text{AppellF1}\left[1 + \frac{1+m}{2}, -m, 1+2(1+m), 1 + \frac{3+m}{2}, \right. \right. \right. \\
& \quad \left. \left. \left. \tan\left[\frac{1}{2}(a+b x)\right]^2, -\tan\left[\frac{1}{2}(a+b x)\right]^2 \right] \sec\left[\frac{1}{2}(a+b x)\right]^2 \tan\left[\frac{1}{2}(a+b x)\right] \right) \right) - \\
& 2 \tan\left[\frac{1}{2}(a+b x)\right]^2 \left(m \left(-\frac{1}{5+m} 2(1+m)(3+m) \text{AppellF1}\left[1 + \frac{3+m}{2}, 1-m, \right. \right. \right. \\
& \quad \left. \left. \left. 1+2(1+m), 1 + \frac{5+m}{2}, \tan\left[\frac{1}{2}(a+b x)\right]^2, -\tan\left[\frac{1}{2}(a+b x)\right]^2 \right] \sec\left[\frac{1}{2}(a+b x)\right]^2 \tan\left[\frac{1}{2}(a+b x)\right] \right. \right. \\
& \quad \left. \left. + \frac{1}{5+m} (1-m)(3+m) \text{AppellF1}\left[1 + \frac{3+m}{2}, 2-m, 2(1+m), 1 + \frac{5+m}{2}, \right. \right. \right. \\
& \quad \left. \left. \left. \tan\left[\frac{1}{2}(a+b x)\right]^2 \right] \right) + 2(1+m) \left(-\frac{1}{5+m} m (3+m) \right. \right. \\
& \quad \left. \left. \text{AppellF1}\left[1 + \frac{3+m}{2}, 1-m, 3+2m, 1 + \frac{5+m}{2}, \tan\left[\frac{1}{2}(a+b x)\right]^2, \right. \right. \right. \\
& \quad \left. \left. \left. \sec\left[\frac{1}{2}(a+b x)\right]^2 \tan\left[\frac{1}{2}(a+b x)\right] \right) + 2(1+m) \left(-\frac{1}{5+m} m (3+m) \right. \right. \right. \\
& \quad \left. \left. \text{AppellF1}\left[1 + \frac{3+m}{2}, 1-m, 3+2m, 1 + \frac{5+m}{2}, \tan\left[\frac{1}{2}(a+b x)\right]^2, \right. \right. \right. \\
& \quad \left. \left. \left. \sec\left[\frac{1}{2}(a+b x)\right]^2 \tan\left[\frac{1}{2}(a+b x)\right] \right) \right)
\end{aligned}$$

$$\left(\left(3 + m \right) \text{AppellF1} \left[\frac{1 + m}{2}, -m, 3 + 2m, \frac{3 + m}{2}, \tan \left[\frac{1}{2} (a + b x) \right]^2, -\tan \left[\frac{1}{2} (a + b x) \right]^2 \right] - 2 \left(m \text{AppellF1} \left[\frac{3 + m}{2}, 1 - m, 3 + 2m, \frac{5 + m}{2}, \tan \left[\frac{1}{2} (a + b x) \right]^2, -\tan \left[\frac{1}{2} (a + b x) \right]^2 \right] + (3 + 2m) \text{AppellF1} \left[\frac{3 + m}{2}, -m, 2(2 + m), \frac{5 + m}{2}, \tan \left[\frac{1}{2} (a + b x) \right]^2, -\tan \left[\frac{1}{2} (a + b x) \right]^2 \right] \right) \tan \left[\frac{1}{2} (a + b x) \right]^2 \right)$$

Problem 189: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \cos[a + b x] \sin[2 a + 2 b x]^m dx$$

Optimal (type 5, 83 leaves, 2 steps):

$$-\frac{1}{b(2+m)} \cos[a + b x] \cot[a + b x] \\ \text{Hypergeometric2F1} \left[\frac{1-m}{2}, \frac{2+m}{2}, \frac{4+m}{2}, \cos[a + b x]^2 \right] (\sin[a + b x]^2)^{\frac{1-m}{2}} \sin[2 a + 2 b x]^m$$

Result (type 5, 173 leaves):

$$\frac{1}{b(-1+4m^2)} i 2^{-1-m} e^{-i(a+b x)} (1 - e^{4i(a+b x)})^{-m} (-i e^{-2i(a+b x)} (-1 + e^{4i(a+b x)}))^m \\ \left((-1 + 2m) \text{Hypergeometric2F1} \left[\frac{1}{4} (-1 - 2m), -m, \frac{1}{4} (3 - 2m), e^{4i(a+b x)} \right] + e^{2i(a+b x)} (1 + 2m) \text{Hypergeometric2F1} \left[\frac{1}{4} (1 - 2m), -m, \frac{1}{4} (5 - 2m), e^{4i(a+b x)} \right] \right)$$

Problem 196: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \csc[c + b x]^2 \sin[a + b x] dx$$

Optimal (type 3, 36 leaves, 4 steps):

$$-\frac{\text{ArcTanh}[\cos[c + b x]] \cos[a - c]}{b} - \frac{\csc[c + b x] \sin[a - c]}{b}$$

Result (type 3, 90 leaves):

$$-\frac{2i \text{ArcTan} \left[\frac{(\cos[c] - i \sin[c]) (\cos[c] \cos[\frac{b x}{2}] - \sin[c] \sin[\frac{b x}{2}])}{i \cos[c] \cos[\frac{b x}{2}] + \cos[\frac{b x}{2}] \sin[c]} \right] \cos[a - c]}{b} - \frac{\csc[c + b x] \sin[a - c]}{b}$$

Problem 201: Unable to integrate problem.

$$\int \sin[a + bx]^2 \sin[c + dx]^n dx$$

Optimal (type 5, 410 leaves, 15 steps):

$$\begin{aligned} & -\frac{1}{2b+d} n 2^{-2-n} e^{i(2a+cn)-i(2b+dn)x+i(n(c+dx))} (1 - e^{2i(c+2idx)})^{-n} \left(\frac{i}{2} e^{-i(c+dx)} - \frac{i}{2} e^{i(c+dx)} \right)^n \\ & \quad \text{Hypergeometric2F1}\left[\frac{1}{2} \left(-\frac{2b}{d} - n\right), -n, \frac{1}{2} \left(2 - \frac{2b}{d} - n\right), e^{2i(c+dx)}\right] + \frac{1}{2b-d} n \\ & \quad \frac{i}{2} 2^{-2-n} e^{i(2a-cn)+i(2b-dn)x+i(n(c+dx))} (1 - e^{2i(c+2idx)})^{-n} \left(\frac{i}{2} e^{-i(c+dx)} - \frac{i}{2} e^{i(c+dx)} \right)^n \\ & \quad \text{Hypergeometric2F1}\left[\frac{1}{2} \left(\frac{2b}{d} - n\right), -n, \frac{1}{2} \left(2 + \frac{2b}{d} - n\right), e^{2i(c+dx)}\right] + \frac{1}{d} n \\ & \quad \frac{i}{2} 2^{-1-n} \left(\frac{i}{2} e^{-i(c+dx)} - \frac{i}{2} e^{i(c+dx)} \right)^n (1 - e^{2i(c+dx)})^{-n} \text{Hypergeometric2F1}\left[-n, -\frac{n}{2}, 1 - \frac{n}{2}, e^{2i(c+dx)}\right] \end{aligned}$$

Result (type 8, 19 leaves):

$$\int \sin[a + bx]^2 \sin[c + dx]^n dx$$

Problem 205: Unable to integrate problem.

$$\int \sin[a + bx]^3 \sin[c + dx]^n dx$$

Optimal (type 5, 600 leaves, 18 steps):

$$\begin{aligned} & \frac{1}{3b-d} n 2^{-3-n} e^{i(3a-cn)+i(3b-dn)x+i(n(c+dx))} (1 - e^{2i(c+2idx)})^{-n} \left(\frac{i}{3} e^{-i(c+dx)} - \frac{i}{3} e^{i(c+dx)} \right)^n \\ & \quad \text{Hypergeometric2F1}\left[\frac{1}{2} \left(\frac{3b}{d} - n\right), -n, \frac{1}{2} \left(2 + \frac{3b}{d} - n\right), e^{2i(c+dx)}\right] - \frac{1}{b-d} n \\ & \quad 3 \times 2^{-3-n} e^{i(a-cn)+i(b-dn)x+i(n(c+dx))} (1 - e^{2i(c+2idx)})^{-n} \left(\frac{i}{3} e^{-i(c+dx)} - \frac{i}{3} e^{i(c+dx)} \right)^n \\ & \quad \text{Hypergeometric2F1}\left[-n, \frac{b-d}{2d}, \frac{1}{2} \left(2 + \frac{b}{d} - n\right), e^{2i(c+dx)}\right] - \frac{1}{b+d} n \\ & \quad 3 \times 2^{-3-n} e^{-i(a+cn)-i(b+dn)x+i(n(c+dx))} (1 - e^{2i(c+2idx)})^{-n} \left(\frac{i}{3} e^{-i(c+dx)} - \frac{i}{3} e^{i(c+dx)} \right)^n \\ & \quad \text{Hypergeometric2F1}\left[-n, -\frac{b+d}{2d}, 1 - \frac{b+d}{2d}, e^{2i(c+dx)}\right] + \frac{1}{3b+d} n \\ & \quad 2^{-3-n} e^{-i(3a+cn)-i(3b+dn)x+i(n(c+dx))} (1 - e^{2i(c+2idx)})^{-n} \left(\frac{i}{3} e^{-i(c+dx)} - \frac{i}{3} e^{i(c+dx)} \right)^n \\ & \quad \text{Hypergeometric2F1}\left[-n, -\frac{3b+d}{2d}, \frac{1}{2} \left(2 - \frac{3b}{d} - n\right), e^{2i(c+dx)}\right] \end{aligned}$$

Result (type 8, 19 leaves):

$$\int \sin[a + bx]^3 \sin[c + dx]^n dx$$

Problem 214: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \sec[c + bx]^2 \sin[a + bx] dx$$

Optimal (type 3, 34 leaves, 4 steps):

$$\frac{\cos[a - c] \sec[c + bx]}{b} + \frac{\operatorname{ArcTanh}[\sin[c + bx]] \sin[a - c]}{b}$$

Result (type 3, 88 leaves):

$$\frac{\cos[a - c] \sec[c + bx]}{b} - \frac{2 \operatorname{i} \operatorname{ArcTan}\left[\frac{(\operatorname{i} \cos[c] + \sin[c]) (\cos\left[\frac{bx}{2}\right] \sin[c] + \cos[c] \sin\left[\frac{bx}{2}\right])}{\cos[c] \cos\left[\frac{bx}{2}\right] - \operatorname{i} \cos\left[\frac{bx}{2}\right] \sin[c]}\right] \sin[a - c]}{b}$$

Problem 227: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \cos[a + bx] \csc[c + bx] dx$$

Optimal (type 3, 27 leaves, 3 steps):

$$\frac{\cos[a - c] \operatorname{Log}[\sin[c + bx]]}{b} - x \sin[a - c]$$

Result (type 3, 58 leaves):

$$\frac{1}{2 b} \left(-2 \operatorname{i} \operatorname{ArcTan}[\tan[c + bx]] \cos[a - c] + \cos[a - c] (2 \operatorname{i} b x + \operatorname{Log}[\sin[c + bx]^2]) - 2 b x \sin[a - c] \right)$$

Problem 228: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \cos[a + bx] \csc[c + bx]^2 dx$$

Optimal (type 3, 35 leaves, 4 steps):

$$-\frac{\cos[a - c] \csc[c + bx]}{b} + \frac{\operatorname{ArcTanh}[\cos[c + bx]] \sin[a - c]}{b}$$

Result (type 3, 90 leaves):

$$-\frac{\cos[a - c] \csc[c + bx]}{b} + \frac{2 \operatorname{i} \operatorname{ArcTan}\left[\frac{(\cos[c] - \operatorname{i} \sin[c]) (\cos[c] \cos\left[\frac{bx}{2}\right] - \sin[c] \sin\left[\frac{bx}{2}\right])}{\operatorname{i} \cos[c] \cos\left[\frac{bx}{2}\right] + \cos\left[\frac{bx}{2}\right] \sin[c]}\right] \sin[a - c]}{b}$$

Problem 231: Result unnecessarily involves complex numbers and more than

twice size of optimal antiderivative.

$$\int \sin[a + bx] \tan[c + bx]^2 dx$$

Optimal (type 3, 44 leaves, 6 steps):

$$\frac{\cos[a + bx]}{b} + \frac{\cos[a - c] \sec[c + bx]}{b} + \frac{\operatorname{ArcTanh}[\sin[c + bx]] \sin[a - c]}{b}$$

Result (type 3, 109 leaves):

$$\begin{aligned} & \frac{\cos[a] \cos[bx]}{b} + \frac{\cos[a - c] \sec[c + bx]}{b} - \\ & - \frac{2 i \operatorname{ArcTan}\left[\frac{(\operatorname{i} \cos[c] + \sin[c]) (\cos\left[\frac{bx}{2}\right] \sin[c] + \cos[c] \sin\left[\frac{bx}{2}\right])}{\cos[c] \cos\left[\frac{bx}{2}\right] - \operatorname{i} \cos\left[\frac{bx}{2}\right] \sin[c]}\right] \sin[a - c]}{b} - \frac{\sin[a] \sin[bx]}{b} \end{aligned}$$

Problem 232: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \sin[a + bx] \tan[c + bx] dx$$

Optimal (type 3, 29 leaves, 3 steps):

$$\frac{\operatorname{ArcTanh}[\sin[c + bx]] \cos[a - c]}{b} - \frac{\sin[a + bx]}{b}$$

Result (type 3, 94 leaves):

$$\begin{aligned} & - \frac{2 i \operatorname{ArcTan}\left[\frac{(\operatorname{i} \cos[c] + \sin[c]) (\cos\left[\frac{bx}{2}\right] \sin[c] + \cos[c] \sin\left[\frac{bx}{2}\right])}{\cos[c] \cos\left[\frac{bx}{2}\right] - \operatorname{i} \cos\left[\frac{bx}{2}\right] \sin[c]}\right] \cos[a - c]}{b} - \\ & - \frac{\cos[bx] \sin[a]}{b} - \frac{\cos[a] \sin[bx]}{b} \end{aligned}$$

Problem 233: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \cot[c + bx] \sin[a + bx] dx$$

Optimal (type 3, 29 leaves, 3 steps):

$$- \frac{\operatorname{ArcTanh}[\cos[c + bx]] \sin[a - c]}{b} + \frac{\sin[a + bx]}{b}$$

Result (type 3, 93 leaves):

$$\frac{\cos[bx]\sin[a]}{b} - \frac{2i\operatorname{ArcTan}\left[\frac{(\cos[c]-i\sin[c])(\cos[c]\cos[\frac{bx}{2}]-\sin[c]\sin[\frac{bx}{2}])}{i\cos[c]\cos[\frac{bx}{2}]+\cos[\frac{bx}{2}]\sin[c]}\right]\sin[a-c]}{b} + \frac{\cos[a]\sin[bx]}{b}$$

Problem 234: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \cot[c+bx]^2 \sin[a+bx] dx$$

Optimal (type 3, 46 leaves, 6 steps) :

$$-\frac{\operatorname{ArcTanh}[\cos[c+bx]\cos[a-c]]}{b} + \frac{\cos[a+bx]}{b} - \frac{\csc[c+bx]\sin[a-c]}{b}$$

Result (type 3, 111 leaves) :

$$-\frac{2i\operatorname{ArcTan}\left[\frac{(\cos[c]-i\sin[c])(\cos[c]\cos[\frac{bx}{2}]-\sin[c]\sin[\frac{bx}{2}])}{i\cos[c]\cos[\frac{bx}{2}]+\cos[\frac{bx}{2}]\sin[c]}\right]\cos[a-c]}{b} + \frac{\cos[a]\cos[bx]}{b} - \frac{\csc[c+bx]\sin[a-c]}{b} - \frac{\sin[a]\sin[bx]}{b}$$

Problem 242: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \cos[a+bx]\sec[c+bx]^2 dx$$

Optimal (type 3, 35 leaves, 4 steps) :

$$\frac{\operatorname{ArcTanh}[\sin[c+bx]\cos[a-c]]}{b} - \frac{\sec[c+bx]\sin[a-c]}{b}$$

Result (type 3, 89 leaves) :

$$-\frac{2i\operatorname{ArcTan}\left[\frac{(i\cos[c]+\sin[c])(\cos[\frac{bx}{2}]\sin[c]+\cos[c]\sin[\frac{bx}{2}])}{\cos[c]\cos[\frac{bx}{2}]-i\cos[\frac{bx}{2}]\sin[c]}\right]\cos[a-c]}{b} - \frac{\sec[c+bx]\sin[a-c]}{b}$$

Problem 248: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \cos[a+bx]\tan[c+bx]^2 dx$$

Optimal (type 3, 46 leaves, 6 steps) :

$$\frac{\text{ArcTanh}[\sin[c + bx] \cos[a - c]]}{b} - \frac{\sec[c + bx] \sin[a - c]}{b} - \frac{\sin[a + bx]}{b}$$

Result (type 3, 111 leaves):

$$\begin{aligned} & -\frac{2 \operatorname{ArcTan}\left[\frac{(\operatorname{i} \cos[c]+\sin[c])\left(\cos\left[\frac{bx}{2}\right] \sin[c]+\cos[c] \sin\left[\frac{bx}{2}\right]\right)}{\cos[c] \cos\left[\frac{bx}{2}\right]-\operatorname{i} \cos\left[\frac{bx}{2}\right] \sin[c]}\right] \cos[a-c]}{b} \\ & -\frac{\cos[bx] \sin[a]}{b}-\frac{\sec[c+bx] \sin[a-c]}{b}-\frac{\cos[a] \sin[bx]}{b} \end{aligned}$$

Problem 249: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \cos[a + bx] \tan[c + bx] dx$$

Optimal (type 3, 30 leaves, 3 steps):

$$-\frac{\cos[a + bx]}{b} - \frac{\operatorname{ArcTanh}[\sin[c + bx]] \sin[a - c]}{b}$$

Result (type 3, 93 leaves):

$$\begin{aligned} & -\frac{\cos[a] \cos[bx]}{b}+ \\ & -\frac{2 \operatorname{ArcTan}\left[\frac{(\operatorname{i} \cos[c]+\sin[c])\left(\cos\left[\frac{bx}{2}\right] \sin[c]+\cos[c] \sin\left[\frac{bx}{2}\right]\right)}{\cos[c] \cos\left[\frac{bx}{2}\right]-\operatorname{i} \cos\left[\frac{bx}{2}\right] \sin[c]}\right] \sin[a-c]}{b}+\frac{\sin[a] \sin[bx]}{b} \end{aligned}$$

Problem 250: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \cos[a + bx] \cot[c + bx] dx$$

Optimal (type 3, 29 leaves, 3 steps):

$$-\frac{\operatorname{ArcTanh}[\cos[c + bx]] \cos[a - c]}{b}+\frac{\cos[a + bx]}{b}$$

Result (type 3, 94 leaves):

$$\begin{aligned} & -\frac{2 \operatorname{ArcTan}\left[\frac{(\cos[c]-\operatorname{i} \sin[c])\left(\cos[c] \cos\left[\frac{bx}{2}\right]-\sin[c] \sin\left[\frac{bx}{2}\right]\right)}{\operatorname{i} \cos[c] \cos\left[\frac{bx}{2}\right]+\cos\left[\frac{bx}{2}\right] \sin[c]}\right] \cos[a-c]}{b}+ \\ & -\frac{\cos[a] \cos[bx]}{b}-\frac{\sin[a] \sin[bx]}{b} \end{aligned}$$

Problem 251: Result unnecessarily involves complex numbers and more than

twice size of optimal antiderivative.

$$\int \cos[a + bx] \cot[c + bx]^2 dx$$

Optimal (type 3, 46 leaves, 6 steps) :

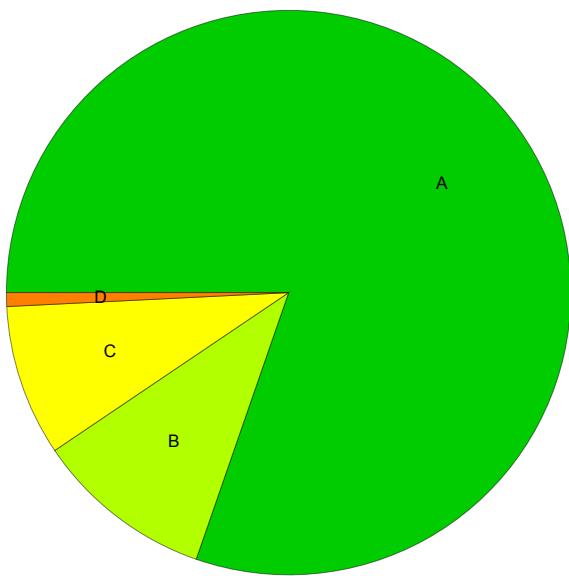
$$-\frac{\cos[a - c] \csc[c + bx]}{b} + \frac{\operatorname{ArcTanh}[\cos[c + bx]] \sin[a - c]}{b} - \frac{\sin[a + bx]}{b}$$

Result (type 3, 112 leaves) :

$$\begin{aligned} & -\frac{\cos[a - c] \csc[c + bx]}{b} - \frac{\cos[bx] \sin[a]}{b} + \\ & \frac{2 \operatorname{ArcTan}\left[\frac{(\cos[c] - i \sin[c]) (\cos[c] \cos\left[\frac{bx}{2}\right] - \sin[c] \sin\left[\frac{bx}{2}\right])}{i \cos[c] \cos\left[\frac{bx}{2}\right] + \cos\left[\frac{bx}{2}\right] \sin[c]}\right] \sin[a - c]}{b} - \frac{\cos[a] \sin[bx]}{b} \end{aligned}$$

Summary of Integration Test Results

254 integration problems



A - 204 optimal antiderivatives

B - 26 more than twice size of optimal antiderivatives

C - 22 unnecessarily complex antiderivatives

D - 2 unable to integrate problems

E - 0 integration timeouts